The Hidden Multi-Line Geometry of Rational Fractions

Resolving Indeterminate Forms and Path Entropy via Extraction Operators

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Introduction & The Core Hypothesis

The Classical View

- Rational functions like $y = \frac{P(x)}{Q(x)}$ exclude points where $Q(x) = 0$.
- Removable singularities are treated as discrete, empty **holes**.

The Relational View

- Since $0 \cdot k = 0$ for any $k \in \mathbb{R}$, we declare $\frac{0}{0} = \mathbb{R}$.
- Without premature cancellation, the denominator condition $Q(x) = 0$ expands into a **complete coexisting geometric line branch**.

The Type Paradox & The Choice Operator

The Type Mismatch

Writing $y = \frac{0}{0}$ equates an **Element** ($y \in \mathbb{R}$) to a **Set** ($\mathbb{R}$). This violates strict programming and algebraic type safety.

The Solution: Entropy-Lowering Operator $\mathcal{E}_d$

We define the indeterminate form structurally as a paired tuple:

$$\frac{0}{0} \triangleq (\mathbb{R}, \mathcal{E}_d) \implies y = \left(\frac{0}{0}\right)[d]$$

where d represents an explicit data descriptor that lowers the topological entropy of the selection space.

Oblique Coordinate Derivations

To keep all intersecting lines fully visible relative to our monitoring axes, we apply an **oblique mixed transformation**:

$$\text{Forward: } z = 2x + y, \quad w = x + 2y$$

$$\text{Inverse: } x = \frac{2z - w}{3}, \quad y = \frac{2w - z}{3}$$

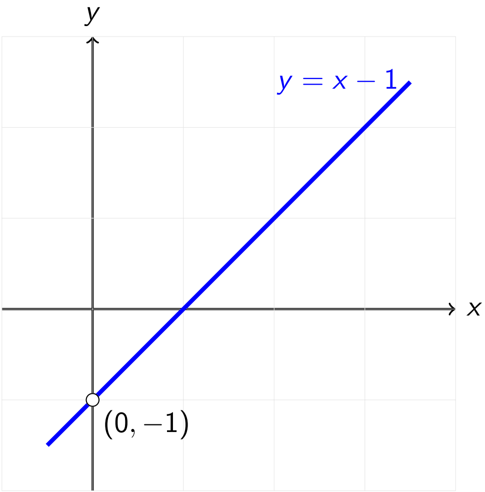
Evaluating $y = \frac{x(x-1)}{x}$ directly in this uncancelled frame yields:

- **Branch 1 ($2z - w \neq 0$):** Simplifies to $z = w + 1 \implies \mathbf{y = x - 1}$
- **Branch 2 ($2z - w = 0$):** Evaluates to $\frac{0}{0} \implies z = \frac{1}{2}w \implies \mathbf{x = 0}$

Both branches intersect perfectly at the vertex $(\mathbf{w}, \mathbf{z}) = (-\mathbf{2}, -\mathbf{1})$.

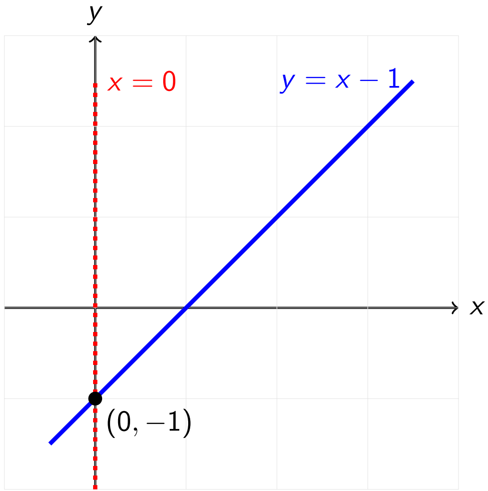
Example 1: Linear Denominator Cancelled

Consider the expression: $y = \frac{x(x-1)}{x}$. Cancelling factor x in both numerator and denominator leaves $y = x - 1, x \neq 0$.



Example 1: Linear Denominator NOT Cancelled

Consider the expression: $y = \frac{x(x-1)}{x}$.



Substituting the inverse transformation formulas directly:

$$\frac{2w-z}{3} = \frac{\left(\frac{2z-w}{3}\right)\left(\frac{2z-w-3}{3}\right)}{\frac{2z-w}{3}}$$

Branch 1 ($2z - w \neq 0$)

Cancelling terms yields:

$$z = w + 1$$

Maps back to: $y = x - 1$

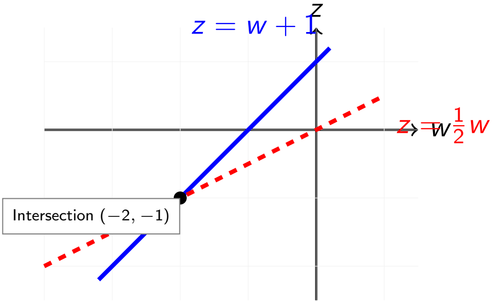
Branch 2 ($2z - w = 0$)

Yields $\frac{0}{0} \implies$ valid line:

$$z = \frac{1}{2}w$$

Maps back to: $x = 0$

Visualizing Linear Solutions: Transformed (z, w) Frame



Extension: Negative Powers of Zero

What happens if the singularity cannot be removed?

$$y = \frac{(x-1)(x-2)}{(x-2)^2} = \frac{x-1}{x-2}$$

- The numerator has a lower multiplicity than the denominator.
- This yields a net **negative power of zero** at $x = 2$.
- Classically, this is an asymptotic boundary where the function breaks completely.

Extension: Negative Powers of Zero

What happens if the multiplicity of the denominator root is higher?

$$y = \frac{(x-1)(x-2)}{(x-2)^2} = \frac{x-1}{x-2} \implies y(x=2) = \mathbb{R} \cdot \infty$$

Asymptotic Absorption

Because $\mathbb{R}$ contains 0, the product $0 \cdot \infty$ still satisfies the underlying relation identity $0 \cdot y = 0$.

Result: The vertical asymptote line $\mathbf{x = 2}$ coexists as a solid geometric branch, intersecting the hyperbola branches continuously at infinity.

Analyzing the Infinite Boundary Relation

Direct substitution into the uncancelled frame yields:

$$\frac{2w - z}{3} = \frac{\left(\frac{2z-w-3}{3}\right)\left(\frac{2z-w-6}{3}\right)}{\left(\frac{2z-w-6}{3}\right)^2}$$

Evaluating at the Singularity ($2z - w - 6 = 0$)

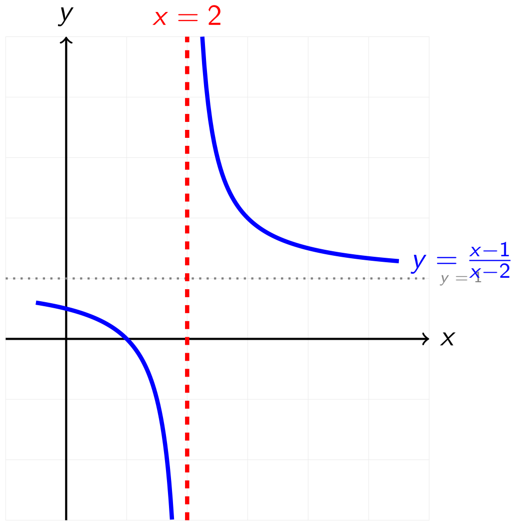
The expression structurally reduces to:

$$y = \frac{\alpha \cdot 0}{0^2} = \left(\frac{0}{0}\right) \cdot \left(\frac{\alpha}{0}\right) \implies \mathbb{R} \cdot \infty$$

Because the set $\mathbb{R}$ includes 0, the product $0 \cdot \infty$ maintains the identity $0 \cdot y = 0$.

Result: The condition is satisfied for any real value. The asymptote line $\mathbf{x = 2}$ coexists as a solid geometric branch, connecting to the hyperbola at infinity.

The Asymptotic Hyperbola Case (x, y) Frame



Complex Manifold Generalization

Fiber Bundles Over $\mathbb{C}$

Let $s \in \mathbb{C}$. Evaluate $W(s) = \frac{s(s-i)}{s}$ at the origin $s = 0$.

$$W(0) = \frac{0}{0} = \mathbb{C}$$

The input point $s = 0$ expands into a complete copy of the complex output plane.

The relational solution space forms a complex manifold union:

1. **Holomorphic Sheet:** Defined by $W = s - i$.
2. **Singular Fiber:** Defined by $\{s = 0\} \times \{W \in \mathbb{C}\}$.

Limits as Path-Dependent Filters

Multi-Valued Derivatives

At a self-intersection point, calculating a derivative yields separate simultaneous answers depending on the chosen trajectory path.

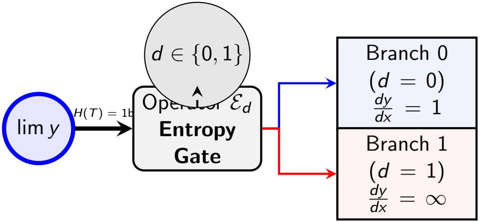
For the intersection vertex of $y = \frac{(x-b)(x-a)}{x-a}$:

- Approaching along the **Functional Line**: $\frac{dy}{dx} = 1$
- Approaching along the **Denominator Line**: $\frac{dy}{dx} = \infty$

The limit operation is structurally a filter, not a single-valued function.

Information Entropy and 1-Bit Routing Gates

- For $N = 2$ intersecting lines, the topological ambiguity creates a structural Shannon entropy of $H(T) = \log_2(2) = 1$ bit.
- Supplying a binary descriptor $d \in \{0, 1\}$ to the extraction operator collapses the conditional entropy to zero ($H(T \mid d) = 0$).



High-Multiplicity Intersections ($N > 2$)

Transformed Axis Singularities

Multiplicity increases when poles track both the native coordinates (x, y) and the transformed axes (z, w) simultaneously:

$$F(z, w) = \frac{(w - z + 1)\left(\frac{2z-w-3}{3}\right)\left(\frac{z}{3}\right)}{\frac{2z-w-3}{3}} = 0$$

At the intersection vertex $(w, z) = (-1, 1) \implies (x, y) = (1, -1)$, three distinct structural lines converge ($N = 3$):

- **Branch 0:** True Solution Line ($y = x - 1$)
- **Branch 1:** Native Denominator Pole ($x = 1$)
- **Branch 2:** Transformed Axis Pole ($2x + y = 0$)

$$\text{Topological Entropy: } H(T) = \log_2(3) \approx 1.585 \text{ bits}$$

The selector argument d scales dynamically to match local node density.

Simultaneous Rational Systems in Engineering

The Problem

An operational network is governed by a rational balance equation and an external linear control constraint:

$$y = \frac{(x-3)(x-1)}{x-1} \quad \text{and} \quad y = -2x + 1$$

Classical Solution

Excludes $x = 1$ as a hole. Yields only **one** point:

$$P_{\text{class}} = \left(\frac{4}{3}, -\frac{5}{3}\right)$$

Relational Solution

Preserves the denominator locus line $x = 1$. Yields a **second** valid state:

$$P_{\text{rel}} = (1, -1)$$

Boundary equilibrium states like P_{rel} are vital for tracking phase-locking or localized stability in real-world control loops.

Relativistic Momentum & Mass-Squared Invariance

The classical formula for relativistic momentum contains a hidden intersection:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

Evaluating exactly at the light barrier for a massless state $(m, v) = (0, c)$ yields:

$$p(0, c) = \frac{0 \cdot c}{\sqrt{0}} \implies \text{Asymmetric Indeterminacy}$$

Squaring the Relation: Physical Insight

Squaring the expression exposes the true underlying Lorentz invariant:

$$p^2 \left(1 - \frac{v^2}{c^2}\right) = m^2 v^2 \xrightarrow{v=c} 0 = m^2 c^2 \implies \mathbf{m^2 = 0}$$

Conclusion: Physical reality operates on **mass-squared invariance (m^2)**. The vertex acts as an entropy gate ($H(T) = 1$ bit), splitting into the subluminal massive branch ($v < c, m^2 > 0$) and the massless photon

Relativistic Momentum: Multiplicity Asymmetry

The classical formula for relativistic momentum contains a hidden intersection:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

Evaluating directly at the light barrier $(m, v) = (0, c)$ yields:

$$p(0, c) = \frac{0 \cdot c}{\sqrt{0}} = \frac{0^1}{0^{1/2}} = 0^{1/2} \to 0$$

The Physical Failure of Linear Fields

Because the multiplicities do not match, a linear evaluation forces $p = 0$. This implies that photons and neutrinos have zero energy and momentum, rendering them physically non-existent.

To form a true indeterminate relation ($0^0 = 0/0$), the system requires matching multiplicities. This symmetry is natively recovered by squaring the relation to reveal **mass-squared invariance (m^2)**:

The Complex Argument of Zero

A real or complex number x mapped in polar coordinates is defined by:

$$x = |x|e^{i\arg(x)} \quad \text{where} \quad \arg(x) = \arctan\left(\frac{\mathrm{Im}(x)}{\mathrm{Re}(x)}\right)$$

Evaluating at the Origin ($x = 0$)

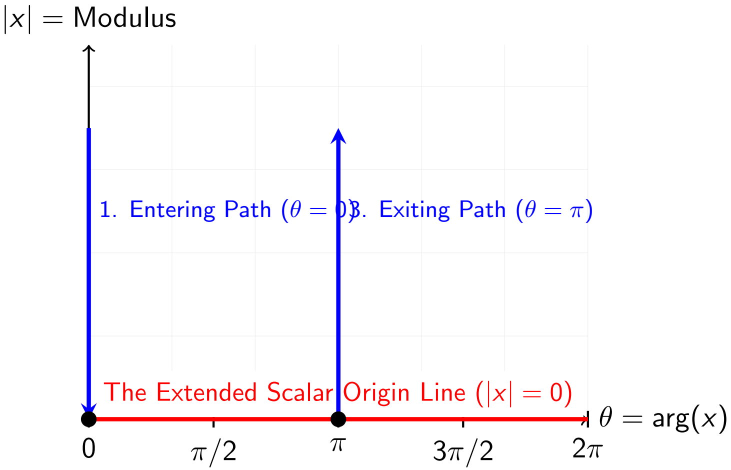
At the scalar origin, both components vanish ($\mathrm{Im}(0) = 0, \mathrm{Re}(0) = 0$):

$$\arg(0) = \arctan\left(\frac{0}{0}\right)$$

Applying our framework: $\frac{0}{0} = \mathbb{R} \implies \arg(0) = \theta \in [0, 2\pi)$

Geometric Insight: In the modulus-argument $(|x|, \theta)$ phase space, zero is not a isolated point. It is a **complete horizontal line** at $|x| = 0$. The data argument d selects the explicit direction as a trajectory crosses the origin.

The Complex Argument of Zero: The Origin in Modulus-Argument Frame



Conclusion

- Reinterpreting $\frac{0}{0} = \mathbb{R}$ uncovers a hidden, multi-line structural geometry in rational functions.
- The type-safe operator $\mathcal{E}_d$ resolves mathematical formatting paradoxes.
- Limits and derivatives at intersection vertices can be cleanly modeled and filtered using discrete 1-bit information keys.