Satori Relaxation of the Internal Family Space in the Spin-Charge-Family Theory

A Non-Symmorphic $P2_13$ Lattice with 26-Cell Polytope Topology

Elia Dmitrieff
SI JAM Labs

The 29th International Workshop
"What Comes Beyond the Standard Models"
10th July, 2026

Context: Connecting Holonomy Masses to Family Space

From Last Session's Talk

- We explored mass squared (m^2) profiles originating purely from **holonomy along a brane** inside a compactified 4D torus (T^4). The brane is produced by relaxation of the **tesseract lattice** following the Satori shift algorithm.
- Proved that vacuum topology regulates external mass parameters without continuous scalar dialing.

The Overlap with Norma's Spin-Charge-Family (SCF) Updates

- The SCF theory unifies generations using the second Clifford algebra $\tilde{\gamma}^a$.
- Traditionally, this leaves states locked inside a flat, stressed **4D hyper-cubic tesseract** $(2^4 = 16$ vertices).
- This high symmetry yields a degenerate 4th family and requires highly engineered VEV configurations within the spin connections ω_{abc}.

The Satori Proposal: Polytope Relaxation

Core Concept

Rather than treating the internal family grid as a rigid tesseract, we apply the **Satori relaxation scheme** for dense 4D space-filling.

- **The Polytope Shift:** Relaxes the hyper-cubic lattice coordinates down to an optimal **26-cell polytope configuration**.
- **Packing Efficiency:** Achieves a 4D structural space-filling envelope that is tighter and more stable than the classic regular 24-cell tessellation.
- **Algebraic Preservation:** The entire underlying foundation of nilpotents, projectors, and $\tilde{\gamma}^a$ definitions is kept completely intact.

Preserving the Foundation: From Grassmann to Projectors

The baseline coordinates are derived from the Grassmann coordinates θ^a and derivatives $\frac{\partial}{\partial\theta^a}$ in the extra-dimensional planes:

$$\gamma^a = \theta^a + \frac{\partial}{\partial\theta^a}, \quad \tilde{\gamma}^a = i\left(\theta^a - \frac{\partial}{\partial\theta^a}\right)$$

For independent coordinate pairs (a, b), we define the standard family commuting structures:

Nilpotents and Projectors Matrix Basis

$$(a\tilde{\tilde{b}})\pm = \frac{1}{2}(\tilde{\gamma}^a \mp i\tilde{\gamma}^b), \quad [a\tilde{\tilde{b}}]\pm = \frac{1}{2}(1 \pm \tilde{\gamma}^a\tilde{\gamma}^b) = \frac{1}{2}(1 \pm 2i\tilde{S}^{ab})$$

The 16 unshifted vertices of the family tesseract map to standard binary signs: $|x_1, x_2, x_3, x_4\rangle \in |\pm, \pm, \pm, \pm\rangle$.

The Satori Shifting Protocol

Because a coordinate flip $+1 \to -1$ covers a metric span of **2**, an exact, symmetric half-shift requires a localized translation vector of magnitude $\|\tilde{\delta}\| = \mathbf{1}$.

Sign Pattern	Shift Vector $\vec{\delta}$	Relaxed Vector Coordinate $\vec{v}_{\text{relaxed}}$
++++	$(0,0,0,1)$	$(1,1,1,2)$
-+++	$(0,1,0,0)$	$(-1,2,1,1)$
+-++	$(0,0,1,0)$	$(1,-1,2,1)$
--++	$(0,1,0,0)$	$(-1,0,1,1) \to$ *Pulls state to center hyperplane*
++-+	$(1,0,0,0)$	$(2,1,-1,1)$
+++-	$(0,0,0,1)$	$(1,1,1,0)$
----	$(0,0,0,1)$	$(-1,-1,-1,0)$

Table: Selected entries of the 16 parity-dependent shifting table.

Symmetry Transition: Down to the Chiral $P2_13$ Lattice

- **Breaking Inversion Symmetries:** The directional constraints of the shift table strip away flat mirror reflections ($\vec{x} \to -\vec{x}$).
- **Arrival at $P2_13$ Topology:** The lattice drops into a non-symmorphic $P2_13$ space group.
- **Screw-Axes Network:** Symmetries are preserved exclusively through 2_1 screw operations (combining a fractional $\Delta = 1$ translation with a 180° planar rotation).

Why the 26-Cell Overrides the 24-Cell

The structural relaxation introduces non-uniform coordinate offsets. The resulting 26-cell boundary packs the extra-dimensional manifold densely, minimizing geometric void volume in the vacuum.

Fate of the Spin Connections Under Satori Relaxation

Do the Spin Connections Survive?

Yes, absolutely. They are not replaced; they are geometrically stabilized from free parameters into structural lattice variables.

For extra-dimensional coordinates $c \in \{5, \ldots, 14\}$, the unified covariant derivative maps vector fields directly onto the **4D scalar (Higgs) sector**:

$$\mathcal{D}_c = \partial_c - \frac{i}{2}\omega_{abc}S^{ab} - \frac{i}{2}\tilde{\omega}_{abc}\tilde{S}^{ab}$$

Simultaneous Fermion-Scalar Coupling

$$\mathcal{L}_{\text{Yukawa}} = \bar{\psi}\gamma^c \left(\partial_c - \frac{i}{2}\omega_{abc}S^{ab} - \frac{i}{2}\tilde{\omega}_{abc}\tilde{S}^{ab}\right)\psi$$

- ω_{abc}: Couples to standard charges (S^{ab}) $\rightarrow$ Electroweak breaking.
- $\tilde{\omega}_{abc}$: Couples to family numbers ($\tilde{S}^{ab}$) $\rightarrow$ Generation mixing.

Crystalline Field Dynamics & Chiral Gauge Splitting

Vacuum Field Equations for $\tilde{\omega}_{abc}$

The non-symmorphic $P2_13$ space group constrains the field via 16 parity-dependent translations $\vec{\delta}_i$, forcing the VEV into localized lattice channels:

$$\left[\mathcal{D}^d\mathcal{D}_d\delta_a{}^{a'}\delta_b{}^{b'}\delta_c{}^{c'} - \mathcal{M}^2_{abc}{}^{a'b'c'}(P2_13)\right]\tilde{\omega}_{a'b'c'}(\vec{x}) = 0$$

Geometric Origin of Left-Right Parity Violation

Integrating over the chiral $P2_13$ compactification volume forces an explicit asymmetry between left- and right-handed effective 4D gauge couplings (g_{eff}):

$$g_{\text{eff}}^{\text{L/R}} \propto \int d^{10}x\, E\, \bar{\psi}_{\text{L/R}}\gamma^\mu \left(\omega_{ab\mu}S^{ab}\right)\psi_{\text{L/R}} \implies \mathbf{g}^{\text{L}}_{\text{eff}} \neq \mathbf{g}^{\text{R}}_{\text{eff}}$$

Parity violation in the weak sector emerges natively from the **structural relaxation of the vacuum itself**.

Eliminating the Fourth Family Topologically

The Long-Standing SCF Tension

The unshifted algebra natively supports 4 families in the visible sector. Suppressing the 4th generation usually requires imposing heavy mass-splitting manually inside the scalar potential.

The Satori / $P2_13$ Solution

- The non-symmorphic screw axes isolate a **unique, asymmetric orientation** during compactification.
- Wavefunctions mapped to this designated coordinate branch fail to achieve periodic closure across the 26-cell boundaries.
- This family branch **topologically decouples** from light gauge sector fields.
- The framework naturally yields exactly the **3 active generations** observed at low energies.

Native Mass Splitting and Wavefunction Distances

In SCF theory, the effective Yukawa mass matrix elements M_{ij} scale inversely with the geometric distance D_{ij} between family components:

$$M_{ij} \propto \exp\left(-\alpha D_{ij}^2\right)$$

Unshifted Tesseract

- Highly uniform.
- Only 4 distinct distances.
- Highly degenerate tree-level masses.

Satori Relaxed Lattice

- Degeneracies lifted.
- **6 unique distances**: $\{2.0, \sqrt{6}, \sqrt{12}, \sqrt{14}, 4.0, \sqrt{22}\}$.
- Asymmetric mass hierarchies $(m_e \ll m_\mu \ll m_\tau)$ emerge natively from the vacuum structure.

Addressing the Graviton Spin-Projection Controversy

The Phenomenon in Recent SCF Updates

Because gravitons are constructed from Clifford-even objects ($\gamma^a\gamma^b$, $\tilde{\gamma}^a\tilde{\gamma}^b$), they can act as active internal rotation operators, causing controversial **spin projection and helicity flips**.

How Satori Relaxation Restricts the Flipping Sector

- **Lattice Discretization:** The non-symmorphic $P2_13$ topology prevents continuous spin-projection rotations in the extra dimensions.
- **Helical Constraints:** Gravitational spin flips are dynamically forced to follow the quantized, rigid paths of the **26-cell screw axes**.
- **Low-Energy Protection:** The energy required to trigger a localized geometric flip scales with the Satori lattice tension, hiding these anomalous transitions from low-energy 4D observers and preserving standard Einstein gravity.

Graviton Commutation Algebra on the Satori Lattice

The family graviton operator is built from the Clifford-even subalgebra:

$$\hat{G}^{ab} = -ic_g \tilde{S}^{ab} = -\frac{1}{4} c_g [\tilde{\gamma}^a, \tilde{\gamma}^b]$$

Let $\hat{T}_i = \{R_i | \vec{\delta}_i\} \in P2_13$ be a Satori non-symmorphic lattice translation. Evaluating the commutator yields:

The Non-Vanishing Commutator Constraint

$$[\hat{G}^{ab}, \hat{T}_i] = -ic_g \left(\tilde{S}^{ab} - (R_i)^a{}_{a'} (R_i)^b{}_{b'} \tilde{S}^{a'b'} \right) \hat{T}_i \neq 0$$

Physical Phenomenon

- Continuous, arbitrary spin-projection and helicity flips are **algebraically blocked** by the lattice.
- Gravitational transitions are rigidly quantized and forced to conform to the discrete orientation of the **26-cell polytope**.

Topological Protection & 5th Family Dark Matter Stability

The Quantum Gravity Paradox

Norma's framework introduces 8 families $(4+4)$. The 5th family is a natural Dark Matter candidate, but standard quantum gravity loops threaten to destabilize it by breaking continuous global symmetries.

The Satori Solution: Crystalline Invariance

- **Immunity to Gravitational Erasure:** The $P2_13$ space group is a discrete spacetime symmetry embedded in the vacuum geometry, which quantum gravity cannot wash away.
- **Screw-Axis Phase Block:** Transitions between the hidden group (5–8) and our visible group (1–4) require navigating uncompensated fractional translations $(\Delta = 1)$.
- **Absolute Stability:** Overlap integrals vanish identically $(\mathcal{M}_{\text{decay}} = 0)$. The 5th family is topologically locked into stable dark matter by the 26-cell boundary.

Satori Corrections to Baryon Asymmetry & Sphalerons

The Mechanism in Norma's Framework

Scalar fields with extra-dimensional indices $t \in \{9, 10, \ldots, 14\}$ carry color charges. Working alongside a right-handed neutrino condensate, they violate baryon number and generate early matter-antimatter asymmetry.

The Cosmic Coincidence Solved Natively

Standard higher-dimensional cosmologies suffer from severe sphaleron dilution, often erasing pre-existing baryon numbers.

- **First-Order Phase Transition:** The non-symmorphic $P2_13$ lattice geometry transforms the electroweak phase transition into a sharp, strongly first-order transition.
- **Sphaleron Shielding:** Topological bubble boundaries structurally lock out and suppress sphaleron wash-out modes, preserving the primordial baryon excess.
- **The $\Omega_{\text{DM}}/\Omega_{\text{baryon}} \approx 5.4$ Ratio:** Because Families 1–4 and Dark Families 5–8 are components of the same 26-cell polytope, their asymmetry generation parameters share a common geometric scale, natively locking the observed cosmic density ratio.

Conclusions and Summary

- **Integration:** The Satori framework supplements the Spin-Charge-Family theory without modifying its pristine algebraic foundations $(\gamma^a, \tilde{\gamma}^a)$.
- **Geometric Progress:** Relaxes the hypercube into a dense 26-cell polytope configuration, connecting nicely with our earlier work on torus holonomies.
- **Phenomenological Gains:** Natively explains the three-generation limit and mass hierarchies through a non-symmorphic $P2_13$ space group.

Thank you! Open for discussion and questions.