29th International workshop
''What comes beyond the standard models''

Dark matter in sphaleron transitions: a general case

By Danila O. Sopin [x,+]
sopin@sfedu.ru, sopindo@mail.ru

x Research Institute of Physics, Southern Federal University, 344090 Stachki 194, Rostov on Don, Russia
+ National Research Nuclear University MEPhI 115409 Moscow, Russia

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Outline

- Introduction
- Balancing of concentrations
- Particular models
  - WTC
  - Sequential generation
- Conclusions

Introduction

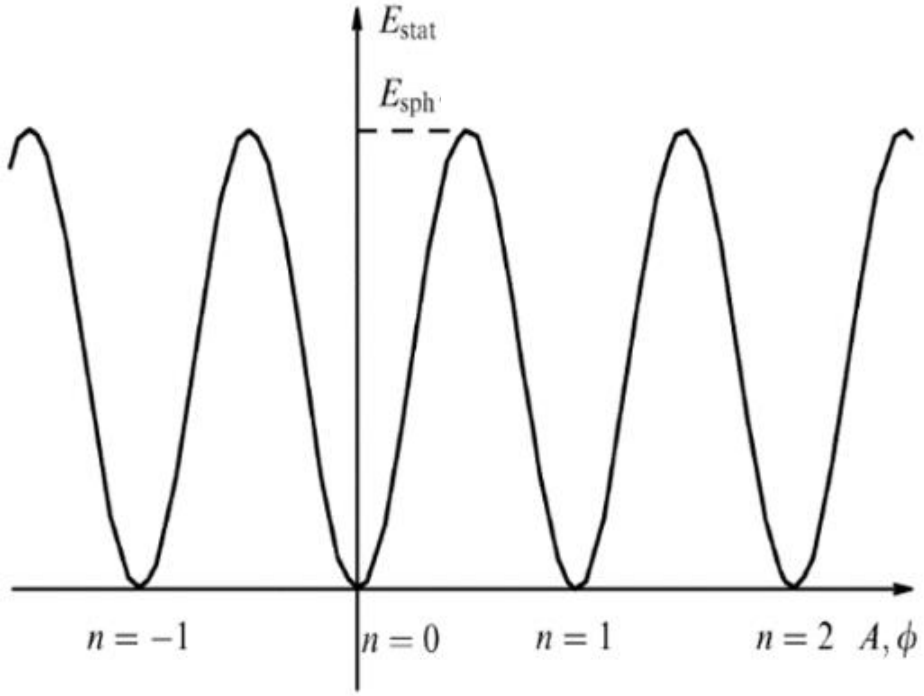
Sphaleron transitions

- **Sphaleron** is a static solution to the electroweak field equations which arises as a consequence of a non-trivial topology of SU(2) group [1].
- Transition between 2 vacua [2]:

$$\mathcal{B} - \mathcal{L} = \text{const} \qquad \mathcal{B} + \mathcal{L} \neq \text{const}.$$

$$\Delta\mathcal{B} = \Delta\mathcal{L} = 3$$

- In SM [3]: $E_{Sph} \approx 9.1\ \text{TeV} \Rightarrow T_* \approx 131\ \text{GeV}$
$T_* < T_c \approx 159\ \text{GeV}$ - temperature of electroweak phase transition



[1] Manton N. S. Topology in the Weinberg-Salam Theory // Phys. Rev. D. — 1983. — Vol. 28. — P. 2019.
[2] Rubakov V. A., Shaposhnikov M. E. Electroweak baryon number non-conservation in the early universe and in high-energy particle collisions // Physics-Uspekhi. — 1996. — Vol. 39, No. 5. — P. 461–502.
[3] Spannowsky M., Tamarit C. Sphalerons in composite and nonstandard Higgs models // Phys. Rev. D. — 2017. — Vol. 95, issue 1. — P. 015006.

Introduction

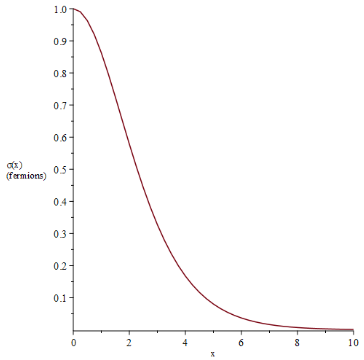
Thermodynamic approach

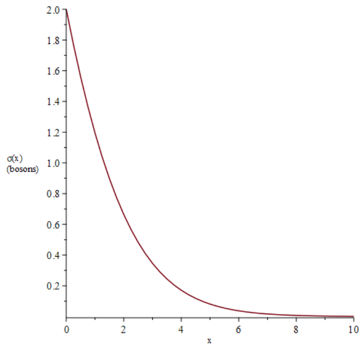
- For baryon number [4]:

$$B = \frac{6}{gT^2}(n_b - n_{\bar{b}}) = \frac{6}{gT^2}\sum_i \frac{1}{6}\, g\, T^3\, \frac{\mu_i}{T}\, \sigma\left(\frac{m_i}{T}\right) =$$

$$= \frac{1}{3}\cdot 3 \cdot (2+\sigma_t)(\mu_{uL} + \mu_{uR}) + \frac{1}{3}\cdot 3\cdot 3\cdot (\mu_{dL} + \mu_{dR}),$$

$$\sigma(z) = \begin{cases} \frac{6}{4\pi^2}\int\limits_0^\infty \mathrm{d}x\, x^2 \left(\cosh\left(\frac{1}{2}\sqrt{x^2+z^2}\right)\right)^{-2}, & \text{for fermions;} \\ \frac{6}{4\pi^2}\int\limits_0^\infty \mathrm{d}x\, x^2 \left(\sinh\left(\frac{1}{2}\sqrt{x^2+z^2}\right)\right)^{-2}, & \text{for bosons.} \end{cases}$$

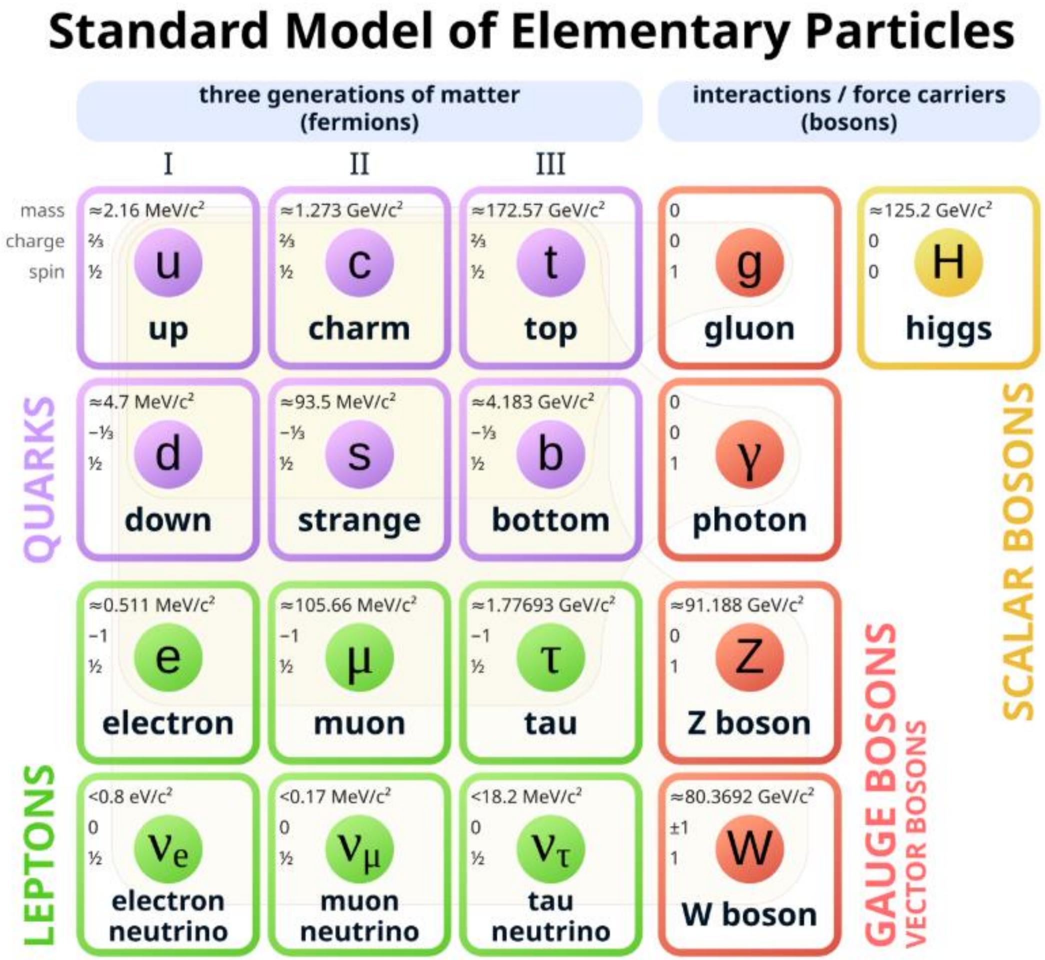




$$\mu_{iR} = \mu_{iL} \pm \mu_0,\ \mu_i = \mu_j + \mu_W,\ \mu_W = \mu_0 + \mu_-$$

[4] Gudnason S. B., Kouvaris C., Sannino F. Dark matter from new technicolor theories // Physical Review D. — 2006. — Vol. 74, no. 9.

Balancing of concentrations

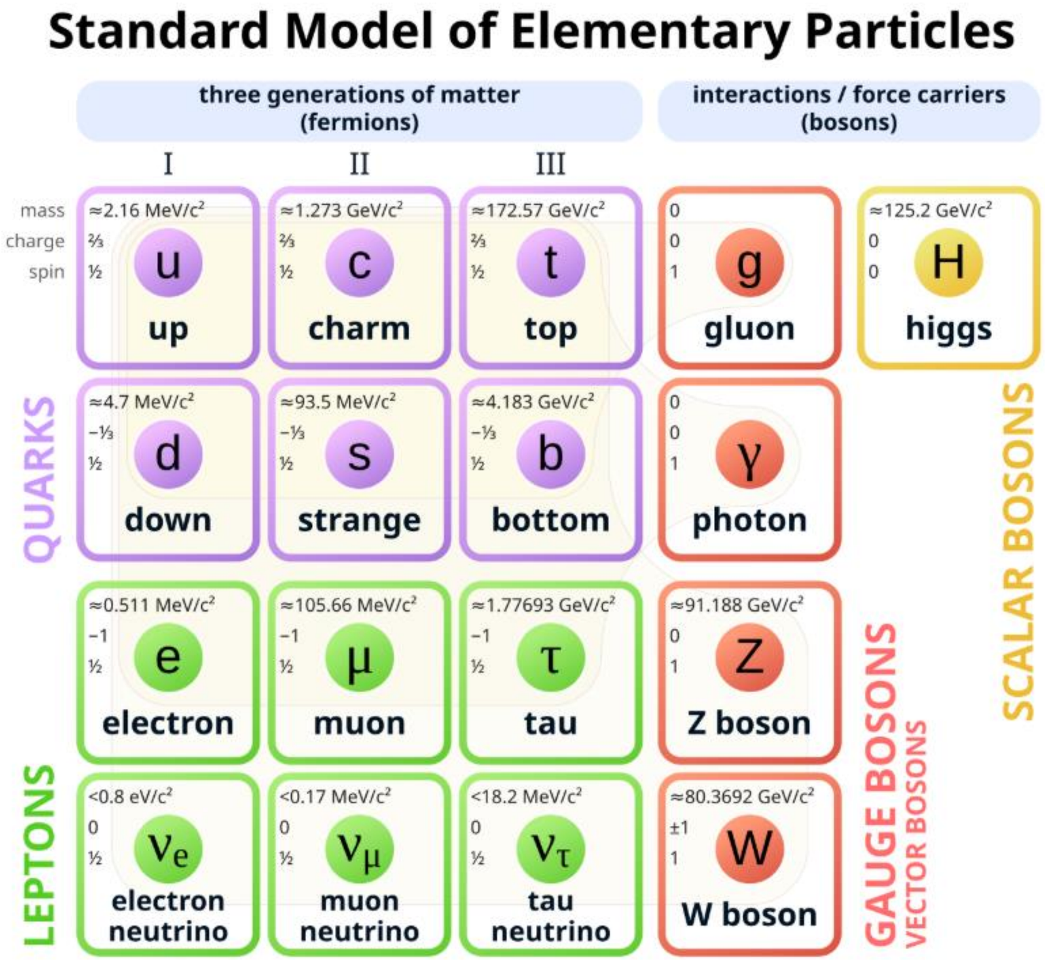


$+ \; X_i$

- Stable
  (at least part of them)
- Interacts with standard SU(2)
  (may carry electric charge)

- It assumes a number new quantum numbers (for example B’ and L’)

Balancing of concentrations



$$+ \quad \begin{pmatrix} U^a \\ D^a \end{pmatrix} \quad \begin{pmatrix} N \\ E \end{pmatrix} \quad + \quad \ldots$$

Balancing of concentrations

$$\begin{pmatrix} a_{11} & 0 & 0 & \cdots & 0 & b_1 \\ 0 & a_{22} & 0 & \cdots & 0 & b_2 \\ 0 & 0 & a_{33} & \cdots & 0 & b_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & a_{n-1,n-1} & b_{n-1} \\ c_1 & c_2 & c_3 & \cdots & cn-1 & d \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_{n-1} \\ \mu_n \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{n-1} \\ 0 \end{pmatrix}$$

The definitions of quantum numbers concentrations

Electric charge conservation

$$\mu_i = \frac{A_i}{a_{ii}} + \frac{b_i}{a_{ii}} \frac{\sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} A_j}{d - \sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} b_j}, \qquad \mu_n = -\frac{\sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} A_j}{d - \sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} b_j}$$

Balancing of concentrations

$$\begin{pmatrix} a_{11} & 0 & 0 & \cdots & 0 & b_1 \\ 0 & a_{22} & 0 & \cdots & 0 & b_2 \\ 0 & 0 & a_{33} & \cdots & 0 & b_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{n-1,n-1} & b_{n-1} \\ c_1 & c_2 & c_3 & \cdots & cn-1 & d \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_{n-1} \\ \mu_n \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{n-1} \\ 0 \end{pmatrix}$$

$$\mu_i = \frac{A_i}{a_{ii}} + \frac{b_i}{a_{ii}} \frac{\sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} A_j}{d - \sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} b_j}, \quad \mu_n = -\frac{\sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} A_j}{d - \sum_{j=1}^{n-1} \frac{c_j}{a_{jj}} b_j}$$

$$\begin{pmatrix} a_{BB} & 0 & 0 & 0 & b_{BW} \\ 0 & a_{LL} & 0 & 0 & b_{LW} \\ 0 & 0 & a_{B'B'}\sigma_{B'} & 0 & b_{B'W}\sigma_{B'} \\ 0 & 0 & 0 & a_{L'L'}\sigma_{L'} & b_{L'W}\sigma_{L'} \\ c_{QB} & c_{QL} & c_{QB'}\sigma_{B'} & c_{QL'}\sigma_{L'} & d_1\sigma_{B'} + d_2\sigma_{L'} + d_3 \end{pmatrix} \cdot \begin{pmatrix} \mu_B \\ \mu_L \\ \mu_{B'} \\ \mu_{L'} \\ \mu_W \end{pmatrix} = \begin{pmatrix} B \\ L \\ B' \\ L' \\ 0 \end{pmatrix}$$

Balancing of concentrations

$$\begin{cases} \sum s_i \mu_i = 0 \\ \sum_{i=3}^{n-1} \frac{m_i}{m_b} \left| \frac{A_i}{B} \right| = \frac{\Omega_{DM}}{\Omega_b} \end{cases}$$

- Condition on chemical potentials arising from Sphaleron transitions
- Sum of densities should not be higher then DM density

Balancing of concentrations

$$\begin{cases} \sum s_i \mu_i = 0 & \text{- Condition on chemical potentials arising from Sphaleron transitions} \\ \sum_{i=3}^{n-1} \frac{m_i}{m_b} \left| \frac{A_i}{B} \right| = \frac{\Omega_{DM}}{\Omega_b} & \text{- Sum of densities should not be higher then DM density} \end{cases}$$

$$(f_{1B}\sigma_{B'} + f_{2B}\sigma_{L'} + f_{3B})B + (f_{1L}\sigma_{B'} + f_{2L}\sigma_{L'} + f_{3L})L+$$
$$+\frac{1}{\sigma_{B'}}(f_{1B'}\sigma_{B'} + f_{2B'}\sigma_{L'} + f_{3B'})B' + \frac{1}{\sigma_{L'}}(f_{1L'}\sigma_{B'} + f_{2L'}\sigma_{L'} + f_{3L'})L' = 0,$$

Exponential increasing

Balancing of concentrations

$$\begin{cases} \sum s_i \mu_i = 0 \\ \sum_{i=3}^{n-1} \frac{m_i}{m_b} \left| \frac{A_i}{B} \right| = \frac{\Omega_{DM}}{\Omega_b} \end{cases}$$

$$+$$

$$\begin{pmatrix} a_{BB} & 0 & 0 & 0 & b_{BW} \\ 0 & a_{LL} & 0 & 0 & b_{LW} \\ 0 & 0 & a_{B'B'}\sigma_{B'} & 0 & b_{B'W}\sigma_{B'} \\ 0 & 0 & 0 & a_{L'L'}\sigma_{L'} & b_{L'W}\sigma_{L'} \\ c_{QB} & c_{QL} & c_{QB'}\sigma_{B'} & c_{QL'}\sigma_{L'} & d_1\sigma_{B'} + d_2\sigma_{L'} + d_3 \end{pmatrix} \cdot \begin{pmatrix} \mu_B \\ \mu_L \\ \mu_{B'} \\ \mu_{L'} \\ \mu_W \end{pmatrix} = \begin{pmatrix} B \\ L \\ B' \\ L' \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\left(1 \pm_{L'} \mp_{B'} \frac{m_{B'}}{m_{L'}} \frac{\sigma_{B'}}{\sigma_{L'}} \gamma'\right) \frac{B'}{B} = -\alpha' \sigma_{B'} \left(\frac{L}{B} + \beta\right) \mp_{L'} \frac{m_b}{m_{L'}} \frac{\sigma_{B'}}{\sigma_{L'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b},$$

$$\frac{L'}{B} = \pm_{L'} \left(\frac{m_b}{m_{L'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b} \mp_{B'} \frac{m_{B'}}{m_{L'}} \frac{B'}{B}\right),$$

- α', β, γ' are ratio of polynomials by σ_i of the same order

$\Rightarrow \alpha', \beta, \gamma' \sim const$ for high masses

- $\alpha = \alpha' \sigma_{B'}, \quad \gamma = \frac{\sigma_{B'}}{\sigma_{L'}} \gamma'$

Balancing of concentrations

$$\left(1 \pm_{L'} \mp_{B'} \frac{m_{B'}}{m_{L'}} \frac{\sigma_{B'}}{\sigma_{L'}} \gamma'\right) \frac{B'}{B} = -\alpha' \sigma_{B'} \left(\frac{L}{B} + \beta\right) \mp_{L'} \frac{m_b}{m_{L'}} \frac{\sigma_{B'}}{\sigma_{L'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b},$$

$$\frac{L'}{B} = \pm_{L'} \left(\frac{m_b}{m_{L'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b} \mp_{B'} \frac{m_{B'}}{m_{L'}} \frac{B'}{B}\right),$$

Exponential suppression

~1 or "slow" exponent

$$\frac{B'}{B} \to \begin{cases} -\alpha' \sigma_{B'} \left(\frac{L}{B} + \beta\right), \\ \pm_{B'} \frac{m_b}{m_{B'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b}, \end{cases} \qquad \frac{L'}{B} \to \begin{cases} \pm_{L'} \frac{m_b}{m_{L'}} \frac{\Omega_{\mathrm{DM}}}{\Omega_b}, & m_{B'} \gg m_{L'} \\ -\alpha' \sigma_{L'} \left(\frac{L}{B} + \beta\right), & m_{B'} \ll m_{L'} \end{cases}$$

- Additional equations simplify these expressions even further:
  - new U(1)-charge conservation law leads to $L' = B'$
  - New practical decay scenario may leads to more complex condition $L' = f(B', B, L)$
- Parameter L/B varies in [5]: $\left|\frac{L}{B}\right| = \left|\frac{n_l - n_{\bar{l}}}{n_b - n_{\bar{b}}}\right| < 6.67 \cdot 10^6.$

[5] Precision big bang nucleosynthesis with improved Helium-4 predictions / C. Pitrou [et al.] // Physics Reports. — 2018. — Vol. 754. — P. 1–66. — ISSN 0370-1573.

Particular models

«Toy» models:

- Minimal walking technicolor [6]
- Model with 4-th sequential generation [7,8]

Both models may face constraints due to their contributions to electroweak observables [9].

	Result
S	0.018 ± 0.095
T	0.04 ± 0.12
U	-0.013 ± 0.086

$$T, U = 0: \quad -0.072 \leq S \leq 0.039;$$

[6] Minimal walking technicolor: Setup for collider physics / R. Foadi [et. al.] // Physical Review D. — 2007. — Vol. 76, no. 5. — ISSN 1550-2368.
[7] Khlopov M. Y. New symmetries in microphysics, new stable forms of matter around us. — 2006. — arXiv: astro-ph/0607048 [astro-ph].
[8] Belotsky K. M., Khlopov M. Y., Shibaev K. I. Composite Dark Matter and its Charged Constituents. — 2006. — arXiv: astro- ph/0604518 [astro-ph].
[9] Review of particle physics / S. Navas [et al.] // Phys. Rev. D. —2024. — Vol. 110, no. 3. — P. 030001.

Particular models

WTC

- 3 technibarions from 2 techniquarks: UU, UD, DD
- 2 technileptons: N, E
- Higgs boson is the composite particle : $\frac{1}{\sqrt{2}}(U\bar{U} + D\bar{D})$.
- Electric charge of techniparticles is not fixed:

UU	$y+1$
UD	y
DD	$y-1$

N	$\frac{-3y+1}{2}$
E	$\frac{-3y-1}{2}$

y	Charges				
	UU	UD	DD	N	E
-5	-4	-5	-6	8	7
-3	-2	-3	-4	5	4
-1	0	-1	-2	2	1
1	2	1	0	-1	2
3	4	3	2	-4	-5
5	6	5	4	-7	-8

[6] Minimal walking technicolor: Setup for collider physics / R. Foadi [et. al.] // Physical Review D. — 2007. — Vol. 76, no. 5. — ISSN 1550-2368.

Particular models

WTC

- Charges of new particles are suppressed by σ and therefore insignificant

$$
\begin{aligned}
Q = 0 = &\left(\left(y_{WTC} + 1\right)\sigma_{UU} + y_{WTC}\sigma_{UD} + \left(y_{WTC} - 1\right)\sigma_{DD}\right)\mu_{UU} - \\
&- \left(\left(3y_{WTC} - 1\right)\sigma_{N} + \left(3y_{WTC} + 1\right)\sigma_{E}\right)\mu_{NL} + \\
&+ 2\left(2\sigma_t + 1\right)\mu_{uL} - 2\mu + \\
&+ \left(y_{WTC}\sigma_{UD} + 2(y - 1)\sigma_{DD} - \left(3y_{WTC} + 1\right)\sigma_{E} - 18\right)\mu_{W}
\end{aligned}
$$

- The signs of charges are more important:
  It is possible to generate excesses of particles with all 4 combinations of
  $\left({}^{TL}/_{B}\,, {}^{TB}/_{B}\right)$: (--), (-+), (+-), (++)

Particular models

WTC

$$\begin{cases} \frac{TB}{B} + \gamma(\sigma_i)\frac{TL}{B} = -\alpha(\sigma_i)\left(\frac{L}{B} + \beta(\sigma_i)\right) \\ m_{UU}\left|\frac{TB}{B}\right| + m_N\left|\frac{TL}{B}\right| = m_b\frac{\Omega_{DM}}{\Omega_b}, \end{cases}$$

$$\Rightarrow \frac{\Omega_b}{\Omega_{DM}}\alpha(\sigma_i)\left(\frac{L}{B} + \beta(\sigma_i)\right) \in \begin{cases} \left[-\frac{1}{m_{UU}}; -\frac{\gamma(\sigma_i)}{m_N}\right]. & (+,+) \\ \left[-\frac{1}{m_{UU}}; \frac{\gamma(\sigma_i)}{m_N}\right], & (+,-) \\ \left[-\frac{\gamma(\sigma_i)}{m_N}; \frac{1}{m_{UU}}\right], & (-,+) \\ \left[\frac{\gamma(\sigma_i)}{m_N}; \frac{1}{m_{UU}}\right]. & (-,-) \end{cases}$$

Particular models

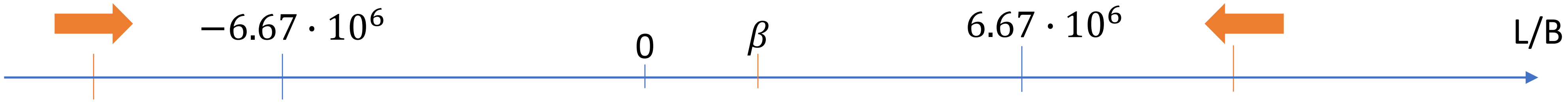
WTC

$$\frac{\Omega_b}{\Omega_{DM}}\alpha(\sigma_i)\left(\frac{L}{B}+\beta(\sigma_i)\right) \in \begin{cases} \left[-\frac{1}{m_{UU}};-\frac{\gamma(\sigma_i)}{m_N}\right]. & (+,+) \\ \left[-\frac{1}{m_{UU}};\frac{\gamma(\sigma_i)}{m_N}\right], & (+,-) \\ \left[-\frac{\gamma(\sigma_i)}{m_N};\frac{1}{m_{UU}}\right], & (-,+) \\ \left[\frac{\gamma(\sigma_i)}{m_N};\frac{1}{m_{UU}}\right]. & (-,-) \end{cases} \quad (1)$$

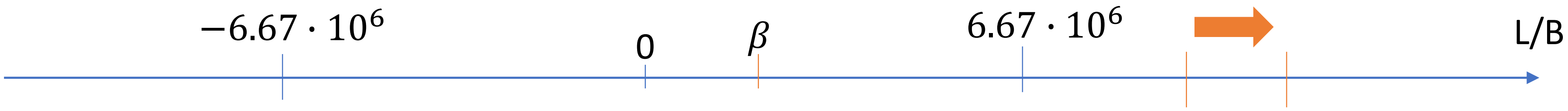
and

$$\left|\frac{L}{B}\right| = \left|\frac{n_l - n_{\bar{l}}}{n_b - n_{\bar{b}}}\right| < 6.67\cdot 10^6. \quad (2)$$

- In cases (-+) and (+-): (2) ∈ (1) if $m_{UU} = m_N > m^{crit} \approx 3.6$ TeV



- In cases (--) and (++): areas do not overlap if $m_{UU} = m_N > m^{crit}$



Particular models

Sequential generation

- New particles: 2 heavy quarks U, D and 2 technileptons N, E
- New gauge U(1) interaction
- Masses of U,D,E ~ TeV, mass of N ~ 50 GeV
- DM: bound state $\bar{U}\bar{U}\bar{U}\bar{N}He$
- U(1)-charge conservation: $FL = FB$!

$$\sum_{i=3}^{n-1} \frac{m_i}{m_b}\left|\frac{A_i}{B}\right| = \frac{\Omega_{DM}}{\Omega_b} \Rightarrow \frac{3m_U + m_E}{m_b}\left|\frac{FB}{B}\right| = \frac{\Omega_{DM}}{\Omega_b}$$

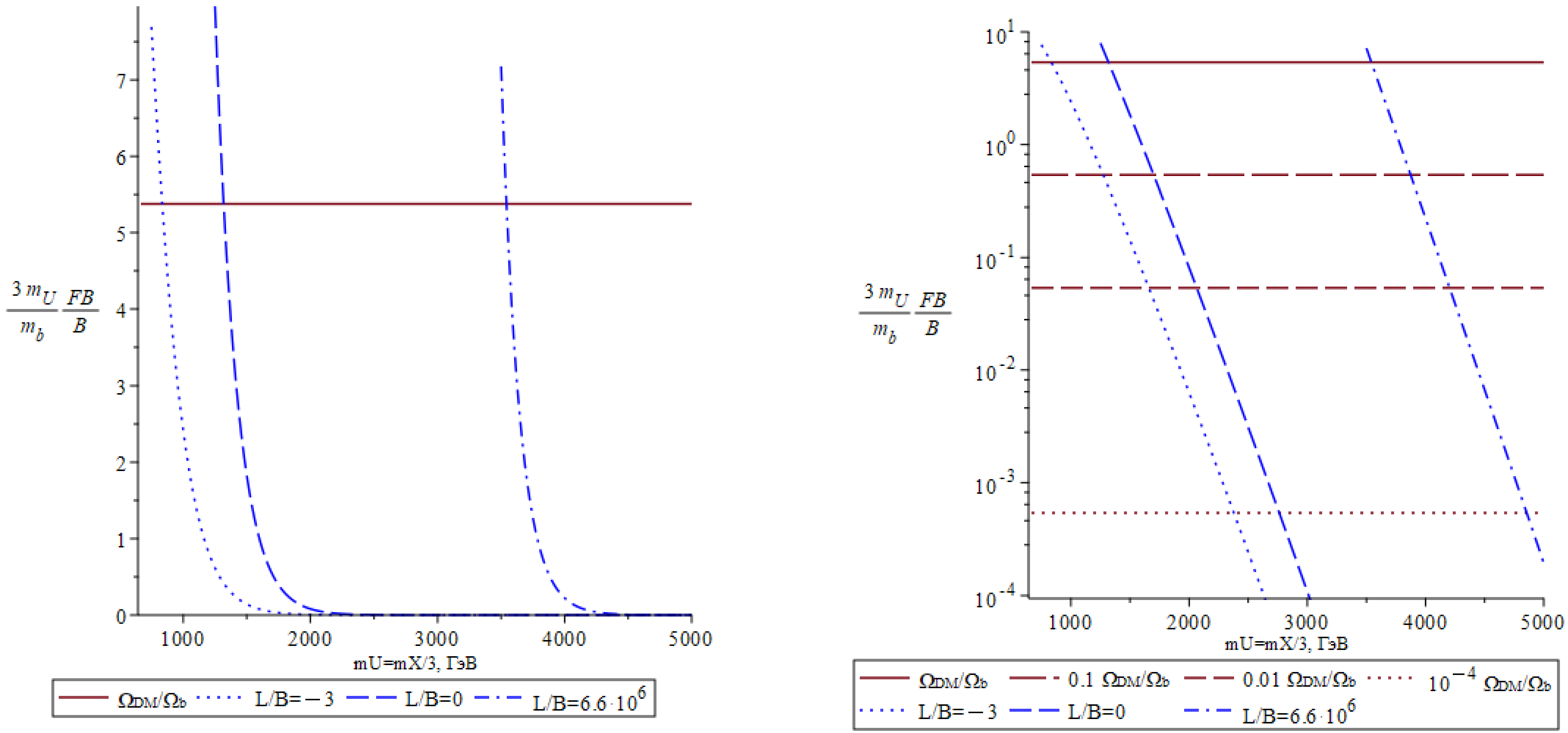
$$\Rightarrow \frac{\Omega_b}{\Omega_{DM}}\alpha'\sigma_U\left(\frac{L}{B}+\beta\right) = \frac{m_b}{3m_U+m_E}\left(1+\frac{\sigma_U}{\sigma_E}\left(1+\frac{m_U}{m_E}(1-\gamma')\right)\right) \quad \textbf{and} \quad \frac{FB}{B} \to -\alpha\left(\frac{L}{B}+\beta\right)$$

$$\to \frac{m_b}{3m_U+m_E}$$

[7] Khlopov M. Y. New symmetries in microphysics, new stable forms of matter around us. — 2006. — arXiv: astro-ph/0607048 [astro-ph].
[8] Belotsky K. M., Khlopov M. Y., Shibaev K. I. Composite Dark Matter and its Charged Constituents. — 2006. — arXiv: astro- ph/0604518 [astro-ph].

Particular models

Sequential generation



$m_{DM} < 11$ TeV

Conclusions

In models with two new global U(1) symmetries (i.e. two baryon-like quantum numbers) and particles interacting with standard electroweak forces, and if $T_*< T_c$ (similar to SM):

- It is possible to find the mass constraints using lepton and baryon asymmetry data only if $(L', B') = (\pm, \pm)$;
- In case $(L', B') = (\pm, \mp)$ only the value of $|\frac{L'}{B'}|$ may be limited (which leads to stronger constraints on $|\frac{L}{B}|$)

The example of the two considered models shows that typical m^{crit} value should be around few TeV.

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By Danila O. Sopin [x,+]
sopin@sfedu.ru, sopindo@mail.ru

x Research Institute of Physics, Southern Federal University, 344090 Stachki 194, Rostov on Don, Russia
+ National Research Nuclear University MEPhI 115409 Moscow, Russia

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