29th International workshop
''What comes beyond the standard models''

Nucleosynthesis in the presence of multiply charged particles

By Danila O. Sopin [x,+]
sopin@sfedu.ru, sopindo@mail.ru

x Research Institute of Physics, Southern Federal University, 344090 Stachki 194, Rostov on Don, Russia
+ National Research Nuclear University MEPhI 115409 Moscow, Russia

2026

Outline

- Introduction
- Constraints on dark atoms
- Neutralization process
  - Recombination-like reactions
    - Ground states
    - Excited states
  - Nuclear capturing
- Concentrations
- Conclusions

Introduction

Dark atom model

- New heavy multiply-charged stable particle X^{-2n} [1,2].
- Electric charge:
  - Negative (to prevent the formation of bound states with electrons)
  - Even (to prevent the overproduction of simple anomalous isotopes $X^{-}He^{+2}$ [3])
- Form neutral bound states with ordinary matter nuclei.
- The simplest (naive) scenario:

$$X^{-2n} + n\,He^{+2} \rightarrow XHe + \gamma$$

[1] Hadronic and Hadron-Like Physics of Dark Matter / V. Beylin [et al.] //Symmetry. — 2019. — Vol. 11, no. 4. — P. 587. — ISSN 2073-8994.
[2] Khlopov M. What comes after the Standard Model? // Prog. Part. Nucl.Phys. — 2021. — Vol. 116. — P. 103824
[3] Belotsky K. M., Khlopov M. Y., Shibaev K. I. Composite Dark Matter and its Charged Constituents // Gravitation and Cosmology. Vol.12. — Moscow : (KPS 06), 2006. — P. 1–7. — e-PrintArchive: astro-ph/0604518.

Constraints on dark atoms

- Minimal mass (strongest, may vary depending on spin)

2n	2	4	6	8	10
m_X, GeV	1060 [4]	1520 [4]	1600 [4]	~2800 [5]	790 [6]

[4] and [6] – anomalously high ionization in ATLAS and MoEDAL
[5] $\bar{X}X \to \gamma\gamma$ - annihilation in LHC

- Maximal mass: unconstrained in general case
BUT: analysis of the sphaleron transitions in particular models indicates

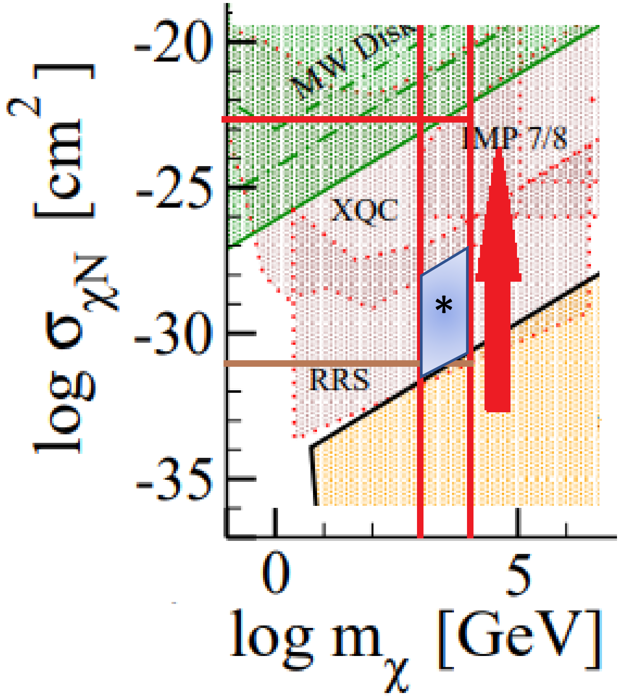
$$m_X^{max} \sim 10 \text{ TeV}$$

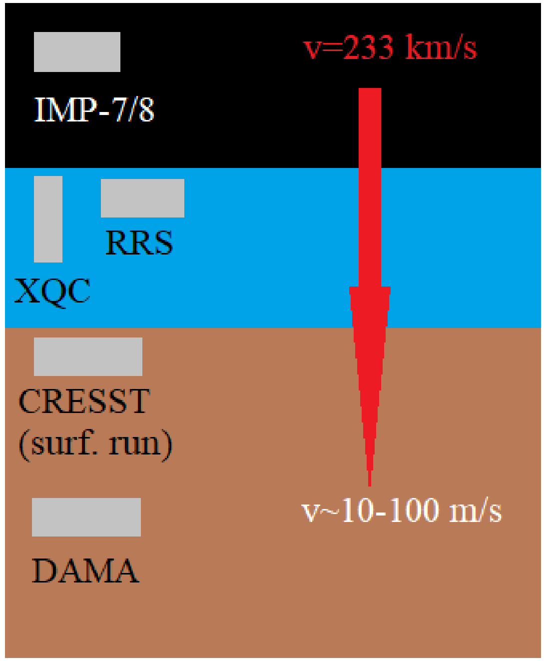
[4] ATLAS Collaboration Search for heavy long-lived multi-charged particles in the full LHC Run 2 pp collision data at $\sqrt{s}$=13 TeV using the ATLAS detector. Phys. Lett. B 2023, 847, 138316.
[5] Altakach, M.; Lamba, P.; Maselek, R. et al. Discovering properties for long-lived multiply charged particles at the LHC. Eur. Phys. J. C. 2022, 82, 848.
[6] Acharya, B. et al. (MoEDAL Collaboration) Search for Highly Ionizing Particles in pp Collisions during LHC Run 2 using the Full MoEDAL Detector. Phys. Rev. Lett. 2025, 134, 071802.

Constraints on dark atoms

- Neutral bound states (dipolar SIMPs)

DM which interact with ordinary matter with $\sigma = \sigma_0 \frac{v_0^2}{v^2}$ avoid the most of SIMP experimental constrains





For X^{-2}He: $\sigma \sim 10^{-31}$ cm^2,
which can be used for the first estimation only because of resonant scattering [10]

$\frac{v_0^2}{v^2} \sim 10^7 - 10^8$, where the thermal velocity was used as a final

*) Region excluded [11] by both RRS and CRESST (surf. run)

[7] Li Y., Liu Z., Xue Y. XQC and CSR constraints on strongly interacting dark matter with spin and velocity dependent cross sections // JCAP— 2023. — Vol. 2023, no. 05. — P. 060. — ISSN 1475-7516.
[8] Dark-matter electric and magnetic dipole moments / K. Sigurdson [et al.] // Phys. Rev. D. — 2004. — Vol. 70, issue 8. — P. 083501.
[9] Mack G. D., Beacom J. F., Bertone G. Towards closing the window on strongly interacting dark matter: Far-reaching constraints from Earth's heat flow // Phys. Rev. D. — 2007. — Vol. 76, no. 4. — ISSN 1550-2368.
[10] Xu X., Farrar G. Resonant scattering between dark matter and baryons: Revised direct detection and CMB limits // Phys. Rev. D. — 2023. — Vol. 107, issue 9. — P. 095028.
[11] Kavanagh B. J. Earth scattering of superheavy dark matter: Updated constraints from detectors old and new // Phys. Rev. D. — 2018. — Vol. 97, no. 12. — ISSN 2470-0029.

Constraints on dark atoms

- Anomalous isotopes

N	n_N^{anom}/n_N		
	1 ТэВ	5 ТэВ	10 ТэВ
H [12]	$3 \cdot 10^{-23}$	$2 \cdot 10^{-21}$	$6 \cdot 10^{-20}$
He [13]	$\sim 10^{-19}$ (Earth); $\sim 10^{-16}$ (Solar system)		
Li [12]	$6 \cdot 10^{-13}$	$9 \cdot 10^{-11}$	$7 \cdot 10^{-10}$
Be [12]	$1 \cdot 10^{-11}$	$1 \cdot 10^{-9}$	$7 \cdot 10^{-9}$
B [12]	$6 \cdot 10^{-14}$	$1 \cdot 10^{-11}$	$8 \cdot 10^{-11}$
C [12]	$8 \cdot 10^{-20}$	$3 \cdot 10^{-17}$	$2 \cdot 10^{-16}$
O [12]	$4 \cdot 10^{-17}$	$4 \cdot 10^{-15}$	$3 \cdot 10^{-14}$
F [12]	$4 \cdot 10^{-15}$	$3 \cdot 10^{-13}$	$2 \cdot 10^{-12}$
Na [14]	$5 \cdot 10^{-12}$		

[12] Search for low-Z nuclei containing massive stable particles / T. K. Hemmick [et al.] // Phys. Rev. D. — 1990. — Vol. 41, issue 7. — P. 2074–2080.
[13] Dick W. J., Greenlees G. W., Kaufman S. L. Search for Anomalous Isotopes of Sodium // Phys. Rev. Lett. — 1984. — Vol. 53, issue 5. — P. 431–434.
[14] Search for Anomalously Heavy Isotopes of Helium in the Earth's Atmosphere / P. Mueller [et al.] // Physical Review Letters. — 2004. — Vol. 92, no. 2. — ISSN 1079-7114.

Neutralization process

(1) $X + N \rightarrow XN + \gamma$

(2) $XN_1 + N_2 \rightarrow XN_3 + N_4/\gamma$

(3) $XN_1 + XN_2 \rightarrow XN_3X + N_4/\gamma$

(1): Similar to hydrogen-like recombination

(2)–(3): Nuclear processes

Neutralization process

Recombination-like reactions

- Stobbe-like recombination rate:

$$\langle \sigma_{rr} v \rangle = \frac{2\sqrt{2\pi}}{3} \frac{Z_N}{Z_X} \frac{1}{m_N^4 r_B^5} \frac{1}{(m_N T)^{3/2}} \cdot \int \frac{1+\eta^2}{\eta^4} \exp\left(-\frac{1}{2m_N r_B^2 T} \frac{1}{\eta^2}\right) \left| \int \rho^2 R_{\varkappa 1}(\rho) \partial_\rho R_{10}(\rho) d\rho \right|^2 d\eta.$$

- Finite size of nucleus:

$$V(\rho) = \begin{cases} -\frac{1}{a}\left(3 - \frac{\rho^2}{a^2}\right), \rho < a \\ -\frac{2}{\rho}, \qquad \rho > a \end{cases}$$

- potential for a homogeneously charged sphere

$\rho = r/r_{Bohr}$ - dimensionless radial variable

$a = r_N/r_{Bohr}$ - radius of a considered nucleus

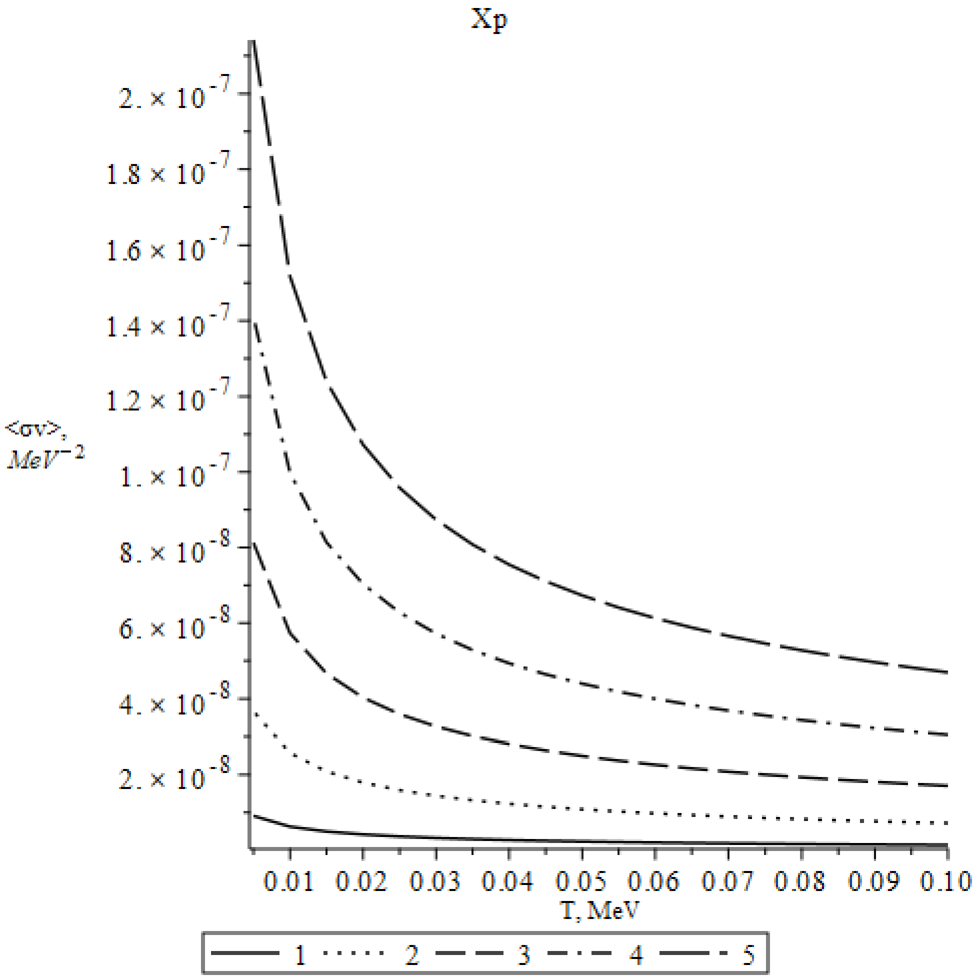
$\eta = Z_1 Z_2 \alpha \sqrt{\frac{\mu_{12}}{2E}} = \frac{1}{k r_B}$ - Sommerfeld parameter

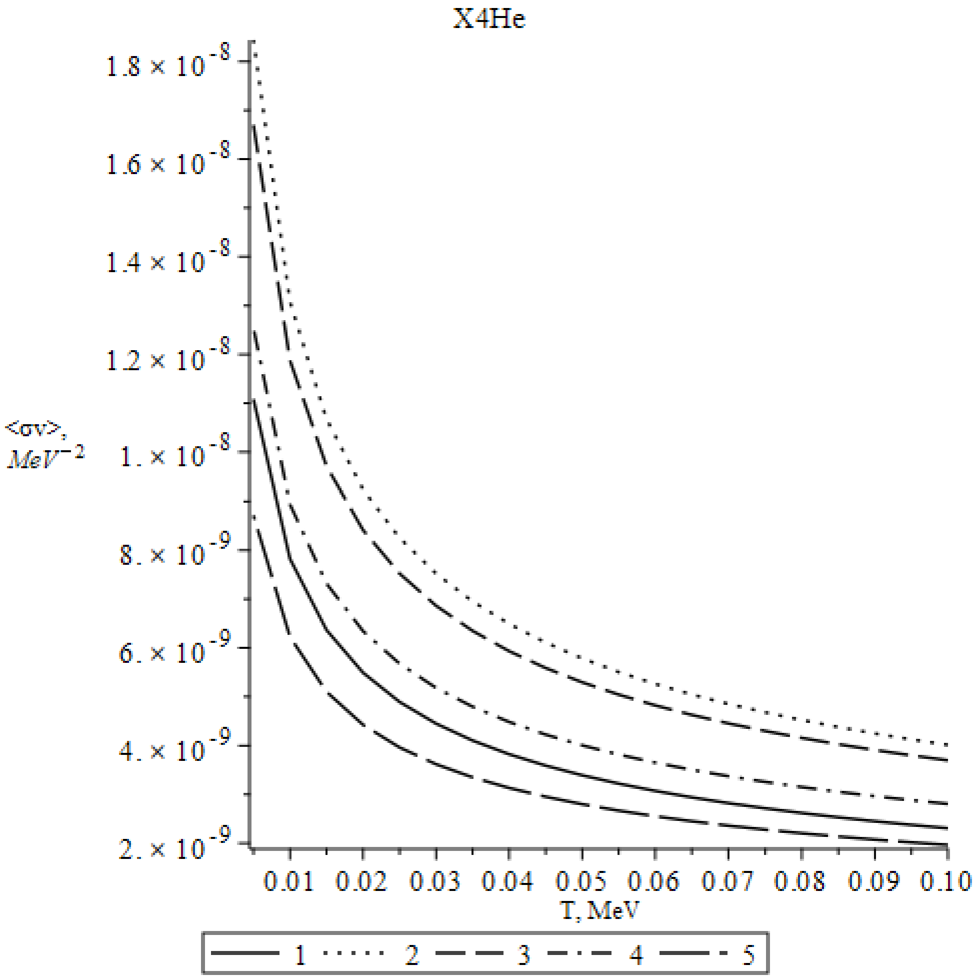
[15] Cahn, R. N.; Glashow, S. L. Chemical Signatures for Superheavy Elementary Particles. Science 1981, 213, 607–611.
[16] Kotelnikov I. A., Milstein A. I. Electron radiative recombination with a hydrogen-like ion // Physica Scripta. — 2019. — Vol. 94, no. 5. — P. 055403. — ISSN 1402-4896.

Neutralization process

Recombination-like reactions

Ground states

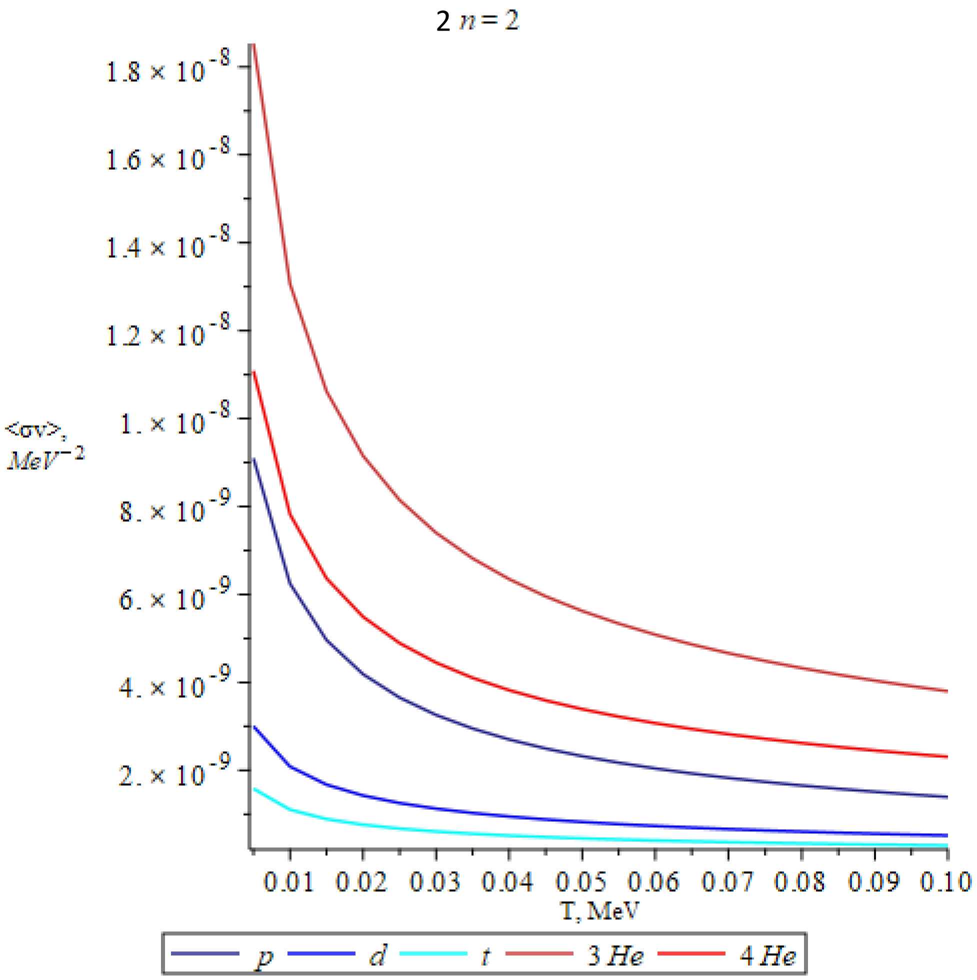


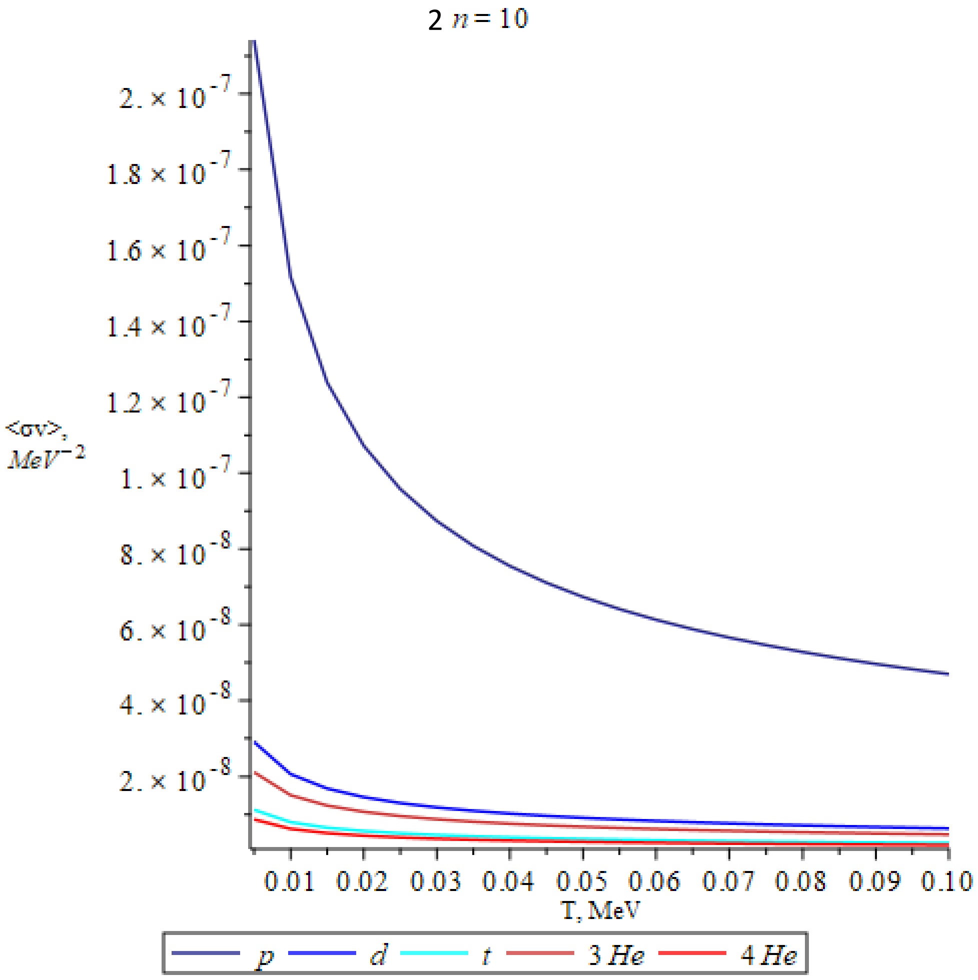


Neutralization process

Recombination-like reactions

Ground states





Neutralization process

Recombination-like reactions

Excited states

- For Coulomb potential:
  - Sum over all transitions [17]

$$\sigma_{rr}^{n_0} = \frac{2^8\pi^2}{3}\frac{Z_N}{Z_X}\frac{1}{m_N^3 r_B}\frac{\eta^6}{(n_0^2+\eta^2)^2} S_{n_0}\left(\frac{4\eta^2}{n_0+\eta}\right)\frac{\exp\left(-4\eta\,\mathrm{arcctg}(\eta/n_0)\right)}{1-\exp\left(-2\pi\eta\right)}$$

  $S_{n_0}(x)$ is a polynomial function obtained by summing over l

  - Semiclassical Kramers formula [16]:

$$\langle\sigma_{rr}v\rangle = \frac{2^5}{3}\sqrt{\frac{\pi}{3}}\frac{Z_N}{Z_X}\frac{1}{m_N^4 r_B^2}\sqrt{\frac{E_{XN}}{T}}\left(\ln\left(\frac{E_{XN}}{T}\right)+\gamma\right)$$

[16] Kotelnikov I. A., Milstein A. I. Electron radiative recombination with a hydrogen-like ion // Physica Scripta. — 2019. — Vol. 94, no. 5. — P. 055403. — ISSN 1402-4896
[17] Katkov V. M., Strakhovenko V. M. Radiative recombination and its application in experiments on electron cooling // Sov. Phys. JETP. — 1978. — Vol. 48, no. 4. — P. 639–643. — ISSN 1269-1277.

Neutralization process

Recombination-like reactions

Excited states

- The first idea:
  if only an amplitude of the rate changes then

$$\langle\sigma v\rangle_{Kramers}^{Finite\ size} = \frac{\langle\sigma v\rangle_{Ground}^{Finite\ size}}{\langle\sigma v\rangle_{Ground}^{Coulomb}} \langle\sigma v\rangle_{Kramers}^{Coulomb}$$

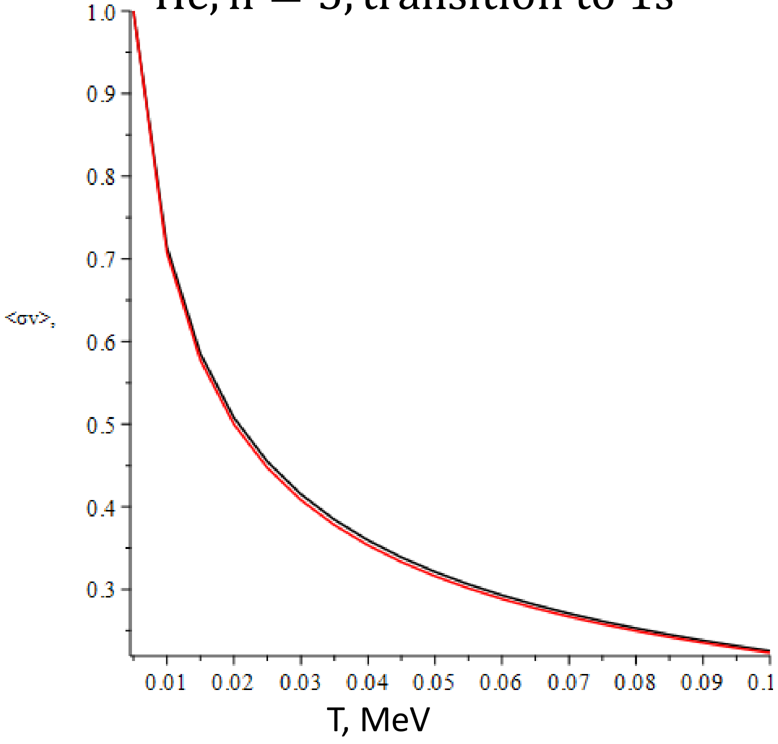
Neutralization process

Recombination-like reactions

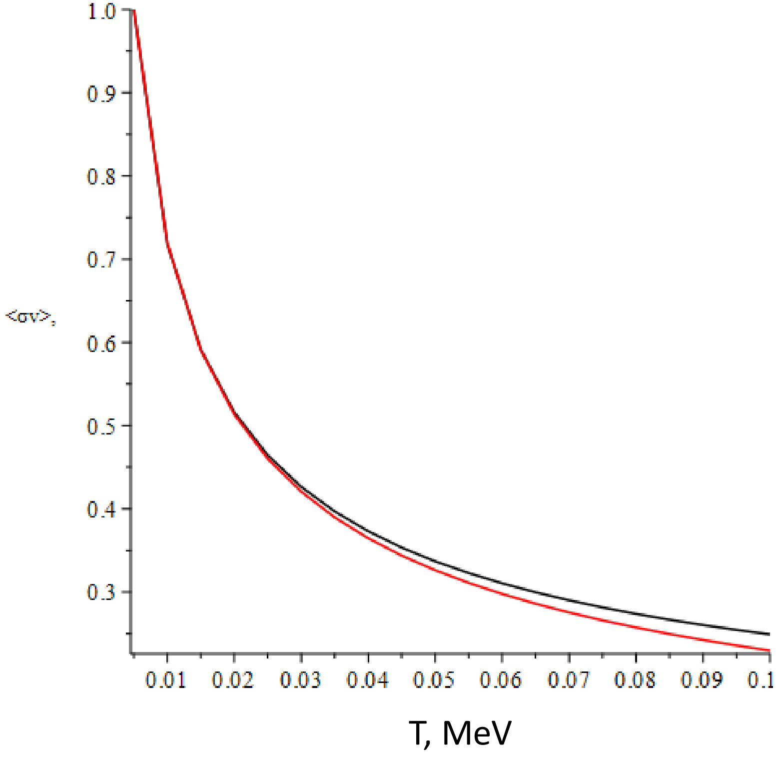
Excited states

- The form of $\langle\sigma v\rangle(T)$ dependence (rescaled to $\langle\sigma v\rangle(T = 5\ \mathrm{keV})$ =1)

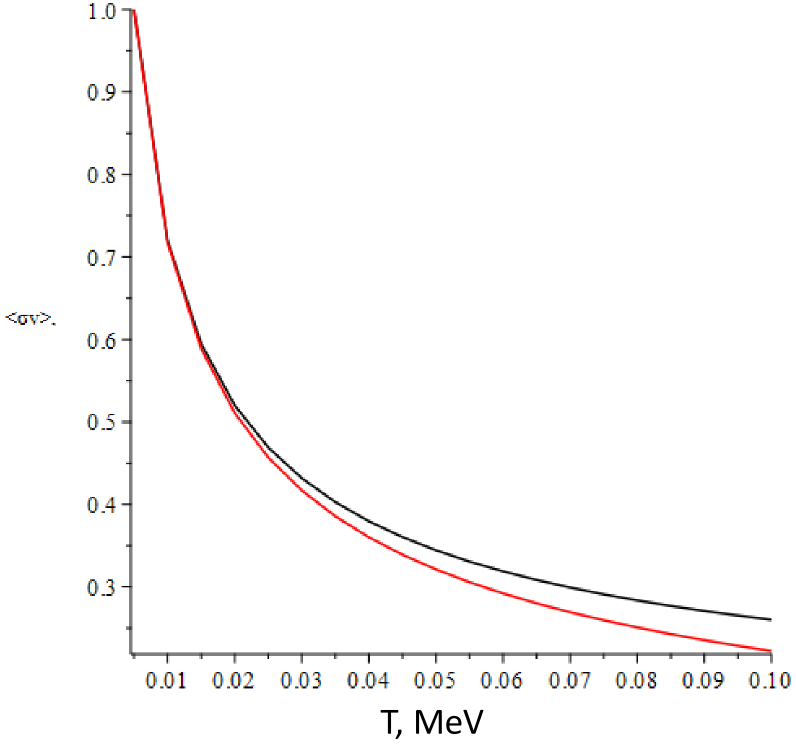
$^{4}\mathrm{He, n = 5, transition\ to\ 1s}$



$^{4}\mathrm{He, n = 5, transition\ to\ 2s}$



$^{4}\mathrm{He, n = 5, transition\ to\ 1s}$



Red line – for Coulomb potential

Black line – potential of the homogeneously charged sphere

Neutralization process

Recombination-like reactions

Excited states

- The first idea:
  if only an amplitude of the rate changes then

$$\langle \sigma v \rangle_{Kramers}^{Finite\ size} = \frac{\langle \sigma v \rangle_{Ground}^{Finite\ size}}{\langle \sigma v \rangle_{Ground}^{Coulomb}} \langle \sigma v \rangle_{Kramers}^{Coulomb}$$

- **BUT:** $\frac{\langle \sigma v \rangle_{Ground}^{Finite\ size}}{\langle \sigma v \rangle_{Ground}^{Coulomb}}$ for (1s,2s,3s) levels are ≈(0.0183, 0.2393, 0.2789);
  for (4s,5s) levels there are >1

Neutralization process

Recombination-like reactions

Excited states

- The first idea:
  if only an amplitude of the rate changes then

$$\langle \sigma v \rangle_{Kramers}^{Finite\ size} = \frac{\langle \sigma v \rangle_{Ground}^{Finite\ size}}{\langle \sigma v \rangle_{Ground}^{Coulomb}} \langle \sigma v \rangle_{Kramers}^{Coulomb}$$

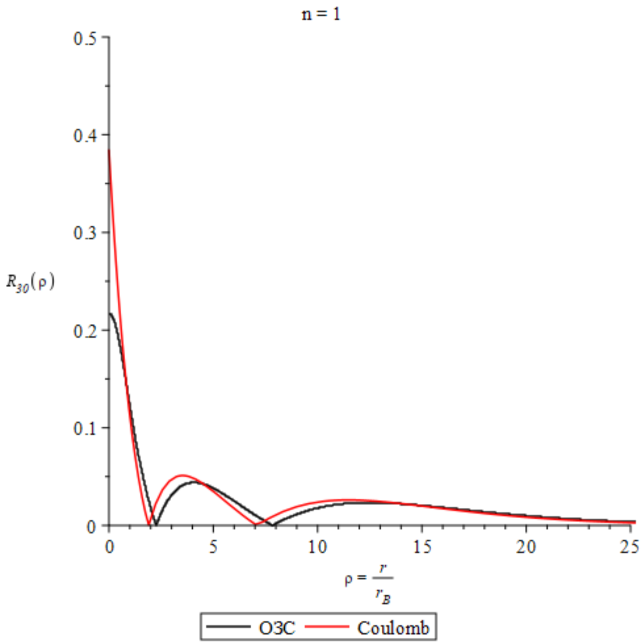
- **BUT:** $\frac{\langle \sigma v \rangle_{Ground}^{Finite\ size}}{\langle \sigma v \rangle_{Ground}^{Coulomb}}$ for (1s,2s,3s) levels are ≈(0.0183, 0.2393, 0.2789);
  for (4s,5s) levels there are >1

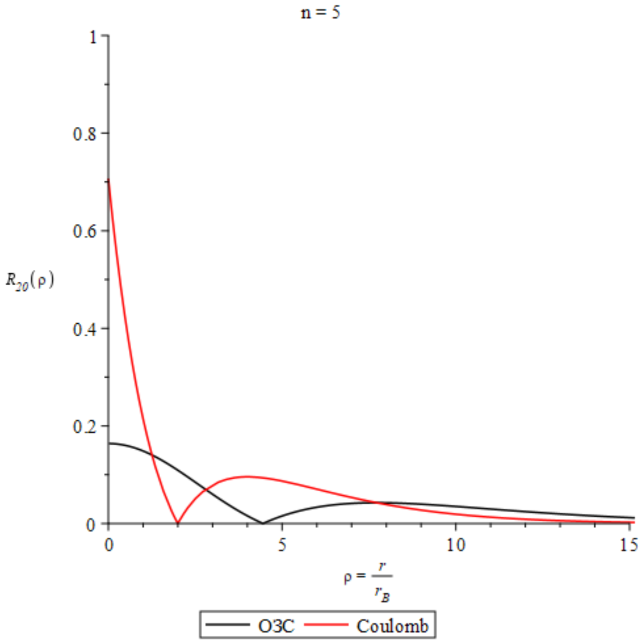
- Future solution: sum over the few levels which should not dissociate at nucleosynthesis temperatures

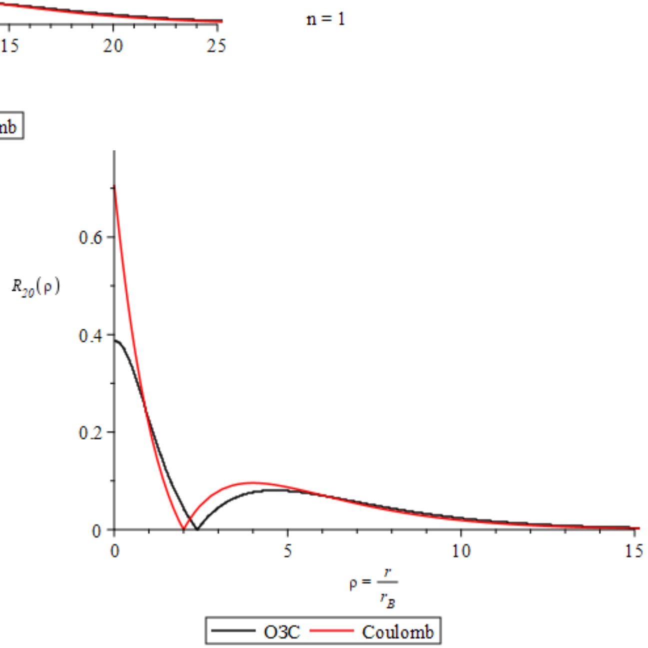
Neutralization process

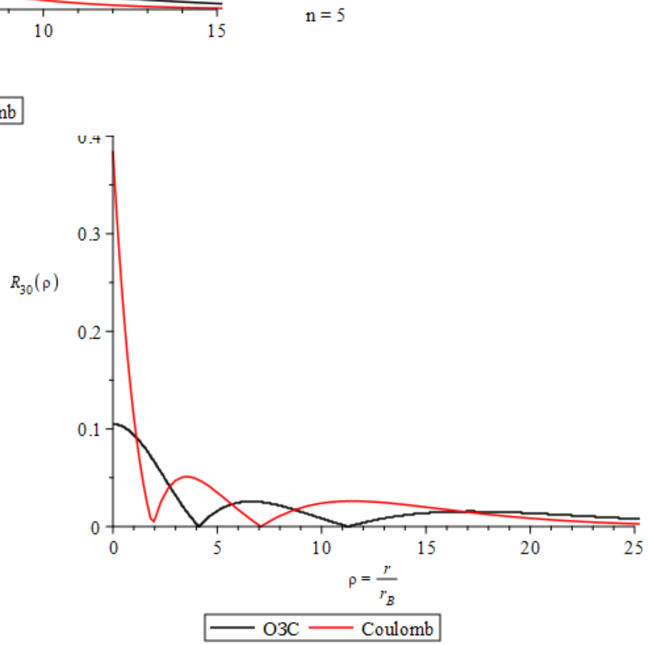
Recombination-like reactions

Excited states









Neutralization process

Nuclear capturing

$$(2)\ XN_1 + N_2 \rightarrow XN_3 + N_4/\gamma$$

- Assumptions:
  - Spin of X $s_X = 0$ ⇒ no new spin-depended interaction
  - All N_i are spherically symmetric* ⇒nuclear deformation due to Coulomb charge shielding is negligible

*preferably it should has a large mass number A

- Then the nuclear data may be obtained from similar reaction

$$N_1 + N_2 \rightarrow N_3 + N_4/\gamma$$

Neutralization process

Nuclear capturing

- General (relativistic) formula for cross section of 2 → 2 reacrion:

$$\sigma_{2\to2} = \int (2\pi)^4 \, \delta^{(4)}(P_i - P_f) \, |M_{fi}|^2 \, \frac{1}{4p_i(\varepsilon_1+\varepsilon_2)} \prod_f \frac{d^3p_f}{(2\pi)^3 2\varepsilon_f} =$$

$$= \int \delta(\varepsilon_i - \varepsilon_f) \left|M_{fi}^{\theta}\right|^2 \frac{1}{16\pi} \frac{1}{p_i(\varepsilon_1+\varepsilon_2)\varepsilon_3\varepsilon_4} \, p_f^2 dp_f ,$$

$$\left|M_{fi}^{\theta}\right|^2 = \int |M_{fi}|^2 \, d\phi_f d(\cos\theta_f)$$

[18] L. D. Landau and E. M. Lifshitz, *Quantum Electrodynamics*, Vol. 4 of *Course of Theoretical Physics* (Butterworth-Heinemann, Oxford, 1982).

Neutralization process

Nuclear capturing

- In zero approximation ($\mathrm{m} \gg E_{kin}, E = m$):

$$\sigma_{4N} \approx \frac{1}{16\pi} \left|M_{fi}^{\theta}\right|^2 \frac{\mu_f \sqrt{\mu_f \mu_i}}{m_1 m_2 m_3 m_4} \sqrt{\frac{E_i + Q}{E_i}}$$

$$\sigma_{3N\gamma} \approx \frac{\sqrt{2}}{32\pi} \left|M_{fi}^{\theta}\right|^2 \frac{\sqrt{\mu_i}}{m_1 m_2 m_3} \frac{E_i + Q}{\sqrt{E_i}}$$

$$\left|M_{fi}^{\theta}\right|^2 = \int \left|M_{fi}\right|^2 d\phi_f d(\cos\theta_f)$$

$$\left|M_{fi}^{\theta}\right|^2 = \left|M_{fi}^{\theta,\mathrm{nuclear}}\right|^2 \cdot |\psi(0)|^2$$

$$|\psi(0)|^2 = \begin{cases} \frac{2\pi\eta}{\exp(2\pi\eta)-1}, & \text{Coulomb repulsion} \\ \frac{2\pi\eta}{1-\exp(-2\pi\eta)}, & \text{Coulomb attraction} \\ 1, & \text{Zero charge} \end{cases}$$

[18] L. D. Landau and E. M. Lifshitz, *Quantum Electrodynamics*, Vol. 4 of *Course of Theoretical Physics* (Butterworth-Heinemann, Oxford, 1982).

Neutralization process

Nuclear capturing

- In zero approximation ($\mathrm{m} \gg E_{kin}, E \approx m$):

$$\sigma_{4N} \approx \frac{1}{16\pi} \left| M^{\theta}_{fi} \right|^2 \frac{\mu_f \sqrt{\mu_f \mu_i}}{m_1 m_2 m_3 m_4} \sqrt{\frac{E_i + Q}{E_i}}$$

$$\sigma_{3N\gamma} \approx \frac{\sqrt{2}}{32\pi} \left| M^{\theta}_{fi} \right|^2 \frac{\sqrt{\mu_i}}{m_1 m_2 m_3} \frac{E_i + Q}{\sqrt{E_i}}$$

$$\left| M^{\theta}_{fi} \right|^2 = \left| M^{\theta,\mathrm{nuclear}}_{fi} \right|^2 \cdot |\psi(0)|^2$$

- Standard parametrization

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-2\pi\eta\right) \quad \text{where} \quad S(E) = a_0 + a_1 E + a_2 E^2$$

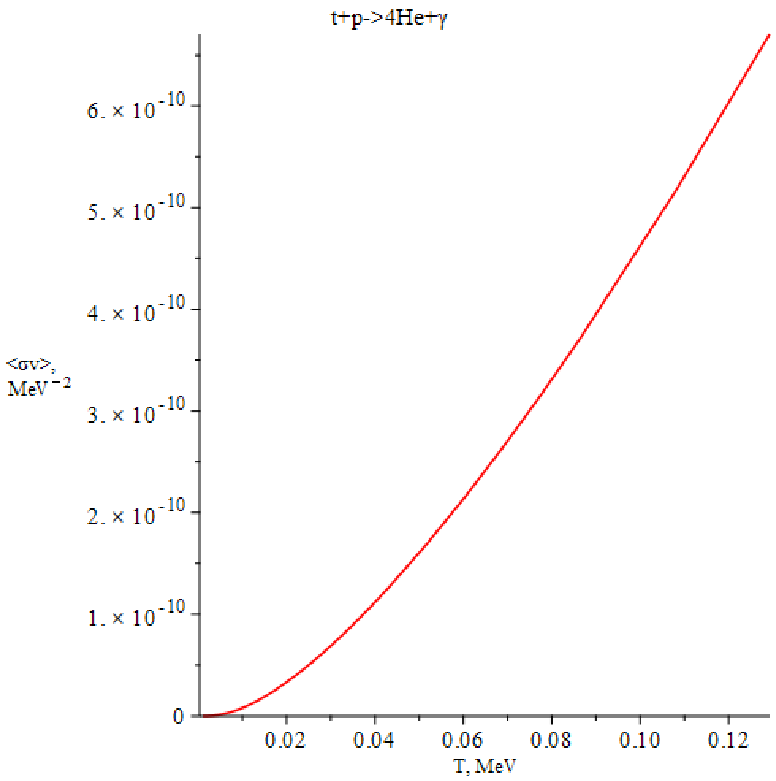
- Finally, $\langle \sigma v \rangle_X = \sqrt{\frac{8}{\pi \mu_i T^3}} \int \frac{\sigma_X}{\sigma} \sigma(E) \exp\left(-\frac{E}{T}\right) E dE$

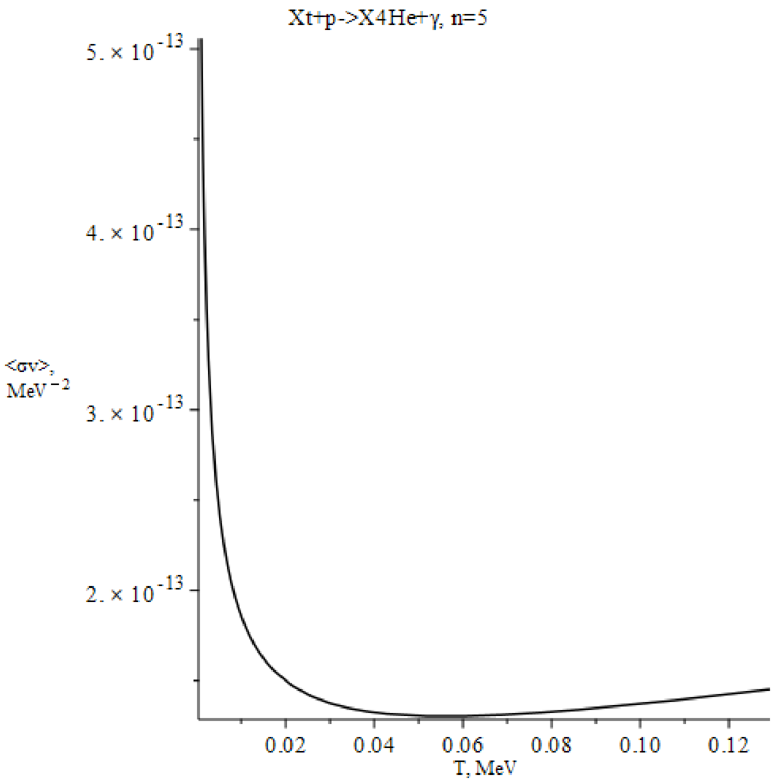
[18] L. D. Landau and E. M. Lifshitz, *Quantum Electrodynamics*, Vol. 4 of *Course of Theoretical Physics* (Butterworth-Heinemann, Oxford, 1982).

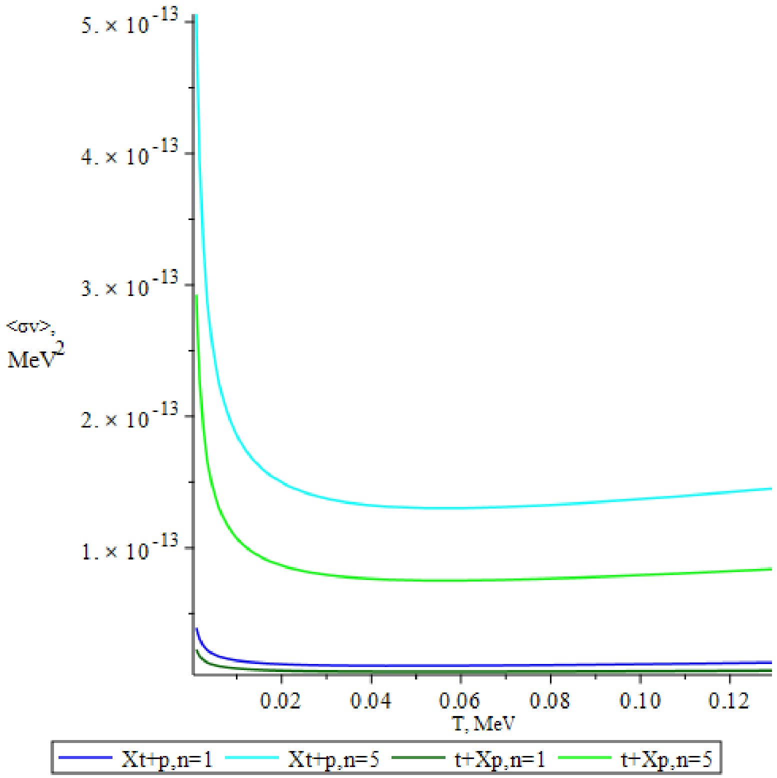
Neutralization process

Nuclear capturing

Reaction: $t + p \rightarrow {}^4He + \gamma$







[19] Nuclear reaction network for primordial nucleosynthesis: a detailed analysis of rates, uncertainties and light nuclei yields/P. D. Serpico [et al.]// JCAP. – 2004. – Vol. 12.

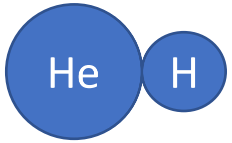
Neutralization process

Nuclear capturing

- High uncertainty is expected in reactions, which include nuclei of

  Li, Be : significant cluster structure, nonspherical form [20,21]

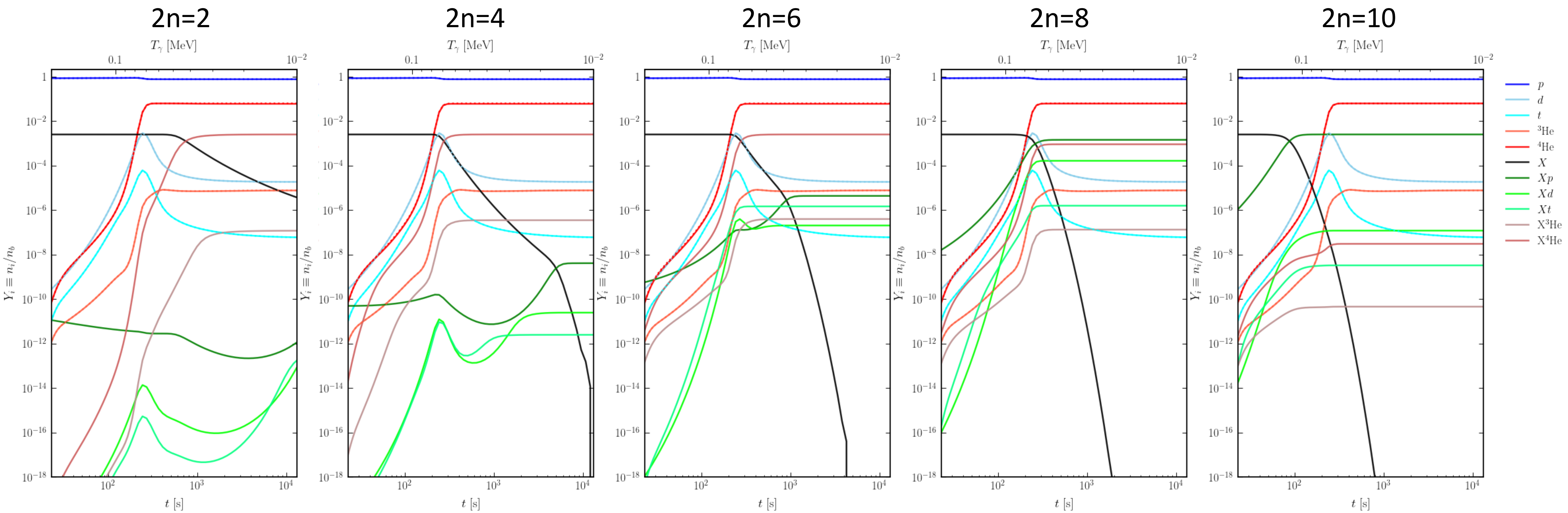
  - Bosonic Hartree–Fock calculations by J. Mwilima for kHe nuclei:
    ^{12}C size changes $\sim 10\%$ with $n = 1 \rightarrow 5$
    ^{8}Be size changes $\sim 150\%$ with $n = 1 \rightarrow 5$
  - All lithium isotopes has “dumbbell” charge distribution [20]
  - Isotopes of beryllium has similar structure



- Stabilization of proton-excessive isotopes?
  Coulomb atomic-like states of dark ion XN^{-q} and single protons?
  What role have multiparticle reactions?

[20] Charge radii and ground state structure of lithium isotopes: Experiment and theory reexamined / W. Nörtershäuser [et al.] // Phys. Rev. C. — 2011. — Vol. 84, issue 2. — P. 024307.
[21] Samarin V. Study of spatial structures in α-cluster nuclei // The European Physical Journal A. — 2022. — Vol. 58.

Concentrations



Only $X + N \to XN + \gamma$ reactions is added.

- $n \leq 3$ the dominant process is **helium** capture
- $n \geq 5$ the dominant process is **proton** capture
- $n = 4$ the processes of **helium** and **proton** capture are comparable

For all n:
the concentration of XH states is high due to the low rate of helium capture.

[22] C. Giovanetti, M. Lisanti, H. Liu, S. Mishra-Sharma, J. T. Ruderman: LINX: A Fast, Differentiable, and Extensible Big Bang Nucleosynthesis Package, arXiv:2408.14538 [astro-ph.CO] (2024)

Conclusions

- Dark atoms avoid the most constraints on SIMP due to the dependance of the cross section on the velocity. Only the region $\sigma{\sim}10^{-28} - 10^{-31}$ is ruled out.
- The rates of the DA formation processes are typically lower than expected from similar and analogous processes due to:
  - Finite size effect for the reaction of the recombination type;
  - High mass of X-particle for the reactions of the nuclear type.
- Processes with lithium and beryllium isotopes require more careful consideration. Such reactions are the key to modified BBN model.
- Scenario of DA formation significantly depends on the charge of X^{-2n}. The role of proton capture processes increases at high values of n.

29th International workshop
''What comes beyond the standard models''

Nucleosynthesis in the presence of multiply charged particles

By Danila O. Sopin [x,+]
sopin@sfedu.ru, sopindo@mail.ru

x Research Institute of Physics, Southern Federal University, 344090 Stachki 194, Rostov on Don, Russia
+ National Research Nuclear University MEPhI 115409 Moscow, Russia

2026

Appendix

Table 1. Properties of $X^{-2n} - p$ bound states

		p				
	q	2	4	6	8	10
Radii	r_N, Fm	0.841				
	r_B, MeV^{-1}	0.073	0.037	0.024	0.018	0.015
	a	0.058	0.115	0.173	0.230	0.288
E_{XN}	E_{XN}^{Coulomb}, MeV	0.100	0.400	0.900	1.599	2.500
	E_{XN}^{Sphere}, MeV	0.100	0.396	0.883	1.550	2.388

Table 2. Properties of $X^{-2n} - d$ bound states

		d				
	q	2	4	6	8	10
Radii	r_N, Fm	2.127				
	r_B, MeV^{-1}	0.037	0.018	0.012	0.009	0.007
	a	0.291	0.583	0.873	1.165	1.456
E_{XN}, MeV	E_{XN}^{Coulomb}	0.200	0.799	1.798	3.196	4.994
	$E_{XN}^{\text{Oscillator}}$	—	—	—	0.604	1.762
	E_{XN}^{Sphere}	0.191	0.701	1.439	2.343	3.370

Table 3. Properties of $X^{-2n} - t$ bound states

		t				
	q	2	4	6	8	10
Radii	r_N, Fm	1.7591				
	r_B, MeV^{-1}	0.0244	0.0122	0.008	0.006	0.005
	a	0.361	0.721	1.082	1.443	1.803
E_{XN}, MeV	E_{XN}^{Coulomb}	0.299	1.197	2.692	4.787	7.479
	$E_{XN}^{\text{Oscillator}}$	—	—	0.288	1.666	3.176
	E_{XN}^{Sphere}	0.280	1.005	2.023	3.242	4.608

Table 4. Properties of $X^{-2n}-{}^3$He bound states

		^3He				
	q	2	4	6	8	10
Radii	r_N, Fm	1.970				
	r_B, 10^{-3} MeV^{-1}	12.20	6.10	4.07	3.05	2.44
	a	0.808	1.615	2.423	3.230	4.037
E_{XN}, MeV	E_{XN}^{Coulomb}	1.196	4.786	10.768	19.142	29.910
	$E_{XN}^{\text{Oscillator}}$	—	1.894	4.767	7.888	11.164
	E_{XN}^{Sphere}	0.978	3.094	5.740	8.690	11.837

Table 5. Properties of $X^{-2n}-{}^4$He bound states

		^4He				
	q	2	4	6	8	10
Radii	r_N, Fm	1.678				
	r_B, 10^{-3} MeV^{-1}	9.19	4.60	3.06	2.30	1.84
	a	0.913	1.826	2.739	3.652	4.566
E_{XN}, MeV	E_{XN}^{Coulomb}	1.588	6.352	14.291	25.406	39.698
	$E_{XN}^{\text{Oscillator}}$	—	2.713	6.195	9.950	13.878
	E_{XN}^{Sphere}	1.256	3.891	7.130	10.708	14.506