Strong primordial inhomogeneities of axion-like field in Einstein–Gauss–Bonnet gravity

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The idea of the present work is coming from the possibility to break or restore the symmetry of the Lagrangian by introduction of the Gauss-Bonnet (GB) scalar. Therefore, we consider the effects of the phase transition induced by the presence of Gauss-Bonnet gravity.

Here we consider Peccei-Quinn (PQ) type of the scalar field, which is coupled to GB scalar via coupling function ξ:

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}R + \partial\Phi\partial\Phi^* - \tfrac{\lambda}{4}(f^2 - 2\Phi\Phi^*)^2 - \tfrac{1}{8}\xi(|\Phi|)R^2_{GB} + \mathcal{L}_{inf}(\chi)\right] . \quad (1)$$

where

$$\xi = \tfrac{1}{6}\alpha\phi^2 = \tfrac{1}{6}\alpha|\Phi|^2,\ \mathcal{L}_{inf}(\chi) = \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\left(1 - e^{-\chi}\right)^2, \quad (2)$$

$$R^2_{\mathrm{GB}} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \quad (3)$$

Potential's minimum

In standart scenario PQ symmetry is initially broken and scalar field Φ has its radial mode fixed:

$$\langle \Phi \rangle = \frac{f}{\sqrt{2}} e^{i\phi/f}, \tag{4}$$

in our model vacuum expectation value $f_{\text{eff}}(t)$ (VEV) is time dependent:

$$f_{\text{eff}}(t) = f\sqrt{1 - \frac{\alpha H^2}{\lambda f^2}(H^2 + \dot{H})} \,. \tag{5}$$

There are three free parameters in the Lagrangian:

$$\lambda = 10^{-6} \;, \quad f = AM \;, \quad \alpha = \frac{b}{M^2} \;, \quad A = 10^3 \;, \quad b = 37.25 \,. \tag{6}$$

Values in (6) are chosen to make Φ subdominant compared to inflaton, what is achieved via condition $\lambda f^4 \ll M^2 M_{Pl}^2$.

VEV' evolution

With the parameters chosen above, VEV evolves as shown below:

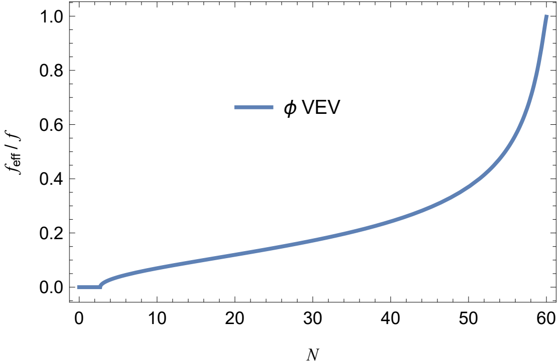

Рис. 1: VEV as a function of e-fold from the start of observable inflation.

Condition $f_{\rm eff}(t) > 0$ is essential for phase transition to occur, but it does not mean it has already happen.

Evolution of the phase

Let us start by considering the full Lagrangian, including the inflaton sector,

$$\sqrt{-g}^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{8}\xi(\phi)R^2_{\mathrm{GB}} - \tfrac{1}{2}(\partial\chi\partial\chi + \partial\phi\partial\phi + \phi^2\partial\theta\partial\theta) - \mathcal{U}(\chi) - \tfrac{\lambda}{4}(f^2-\phi^2)^2\,. \quad (7)$$

Background equation for $\bar{\theta}$ is particularly simple,

$$\ddot{\bar{\theta}} + \left(3H + 2\frac{\dot{\phi}}{\phi}\right)\dot{\bar{\theta}} = 0\;, \quad (8)$$

as there is no potential for it during inflation. Provided that

$$\left|\frac{2\dot{\phi}}{3H\phi}\right| = \left|\frac{2\phi_{,N}}{3\phi}\right| \ll 1\;, \quad (9)$$

Perturbations of the phase

By splitting the phase into two parts:

$$\theta(t,x) = \theta_c(t) + \theta_q(t,x) \ , \tag{10}$$

we can write down equation governing the perturbations of the phase (after Fourier transformation):

$$\delta\ddot{\theta}_k + 3H\delta\dot{\theta}_k + \left(\frac{k^2}{a^2} + 3\phi^2\dot{\bar{\theta}}^2\right)\delta\theta_k \simeq 0 \ . \tag{11}$$

Or, in terms of $u_k \equiv a\phi\delta\theta_k$ and the e-fold time,

$$u_{k,NN} + u_{k,N} + \left[\frac{k^2}{a^2H^2} - 2 + 3\phi^2\bar{\theta}^2_{,N}(N_{\rm in})e^{6(N_{\rm in}-N)}\right]u_k \simeq 0 \ , \tag{12}$$

which is Mukhanov-Sasaki equation.

Evolution of the field Φ

Initially the scalar field is localized around zero and angular degree of freedom is not defined at that quantum stage.

At the moment when $f_{\text{eff}}(t) > 0$, the scalar field starts to roll down.

We define the moment of classicalization of the field via numerical solution to the Langevin equation:

$$\phi_{,N} \simeq -\frac{V_{\text{eff},\phi}}{3H^2} + \Xi_\phi \simeq -\frac{m_{\text{eff}}^2}{3H^2}\phi + \Xi_\phi \ , \tag{13}$$

which yields:

$$\langle\phi^2\rangle_{,N} \simeq -\frac{2m_{\text{eff}}^2}{3H^2}\langle\phi^2\rangle + \left(\frac{H}{2\pi}\right)^2 \ . \tag{14}$$

We define the classicalization criteria in a heuristic way:

$$\partial_{,N}\langle\phi^2\rangle = 10 \times H^2/(4\pi^2) \ . \tag{15}$$

Classical evolution of ϕ

By solving Langevin equations, we can set initial conditions for classical equation of motion for the radial part of the field.

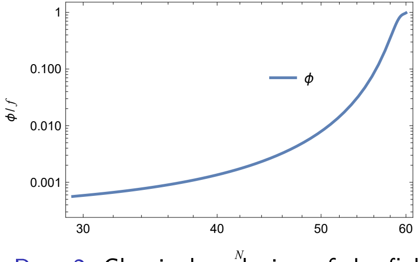

Рис. 2: Classical evolution of the field ϕ.

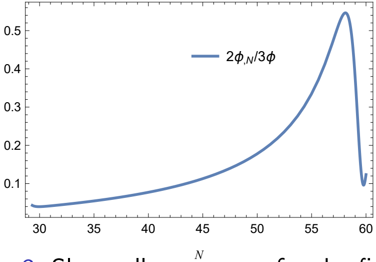

Рис. 3: Slow-roll parameter for the field ϕ.

Power spectrum of the phase

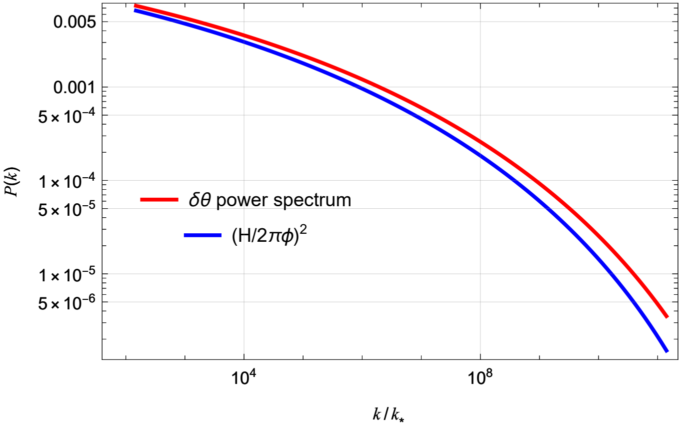

Рис. 4: Power spectrum of the phase. Here $k_*^{-1} \approx 0.4\,Au$ — the scale corresponding to the start of the classical motion.

Given with slow rolling of the ϕ it is easy to show:

$$P_{\delta\theta}(k) = \frac{k^3}{2\pi^2}|\delta\theta_k|^2 \simeq \frac{H^2}{4\pi^2\phi^2}, \tag{16}$$

which provides link between these two approaches ϕ.

Possible cosmological scenario I

Let us consider simple condition for oscillations start:

$$3H = m_{ALP} = \Lambda^2/f, \tag{17}$$

which implies the temperature at the start of the oscillations T_{osc} is found as follows (in reduced Planck units):

$$3 \cdot 1.66\sqrt{g_*(T_{osc})}\,T_{osc}^2 \approx \Lambda^2/f, \tag{18}$$

which implies:

$$T_{osc} \approx \frac{\Lambda}{\sqrt{5f\sqrt{g_*(T_{osc})}}}. \tag{19}$$

Possible cosmological scenario II

Modern ALP density could be estimated as follows:

$$\rho_{ALP,0} = \rho_{ALP,i} \cdot \left(\frac{a_{osc}}{a_0}\right)^3 = \rho_{ALP,i} \cdot \left(\frac{T_0}{T_{osc}}\right)^3, \tag{20}$$

where modern temperature $T_0 \approx 2.6 \cdot 10^{-13} GeV$.
Initial ALP density could be estimated as phase averaged:

$$\rho_{ALP,i} =< U(\theta) >= \Lambda^4 < 1 - \cos\theta >= \Lambda^4, \tag{21}$$

From expression (20):

$$\rho_{ALP,0} \approx 2.8 \cdot 10^{-49} \cdot \frac{\Lambda(GeV)}{10^{-10} GeV} GeV^4. \tag{22}$$

Possible cosmological scenario III

Taking $\Lambda = 10^{-10} GeV$ we obtain:

$$\rho_{ALP,0} \approx 2.8 \cdot 10^{-49} GeV^4 \tag{23}$$

Modern critical density is

$$\rho_0 = \frac{3H_0^2}{8\pi G} \approx 8 \cdot 10^{-47} GeV^4. \tag{24}$$

Given with $\Lambda \sim 10^{-10} GeV$, $f \sim 10^{16} GeV \to m_a \sim 10^{-27} eV$. Therefore — ultra-light fuzzy dark matter. Constrained — $\Omega_a \leq 1\%$.
Note: Distribution of initial phase probably allows to form domain wall network surrounded by strings. This is the next step in the analysis.
During cosmological inflation phase could fluctuate.

Possible cosmological scenario IV

Assume that these fluctuations would enter Hubble horizon during RD, then size of the fluctuations after entering horizon are as follows:

$$r(N) = \frac{e^{2(N_{inf} - N)}}{2HN_{inf}}. \tag{25}$$

Size of the fluctuation allows to estimate its mass:

$$r(N) = \sqrt{\frac{m}{4\pi\sigma}} = \frac{e^{2(N_{inf} - N)}}{2HN_{inf}}. \tag{26}$$

After horizon re-entry a closed domain wall typically starts to shrink until its radius reaches its thickness $d = f/(2\Lambda^2)$. Therefore, to form a black hole it needs to satisfy

Possible cosmological scenario V

$d < r_s$, where $r_s \equiv m/(2\pi)$ is its Schwarzschild radius (in reduced Planck units). By using (26) we can write the mass of the wall (upon horizon re-entry) as

$$m(N) = \frac{4\pi\Lambda^2 f}{H_{inf}^2 N_{inf}^2} e^{N_{inf}-N} \ . \tag{27}$$

Here N corresponds to the forward e-fold number during inflation (taking $N = 0$ at the start of observable inflation, and $N = N_{inf}$ at the end), at which the corresponding ALP fluctuation is generated. Hence, from the collapse condition $d < r_s$ we obtain

$$N < N_{inf} - \frac{1}{2} \ln\left(\frac{H_{inf} N_{inf}}{\sqrt{2}\Lambda^2}\right) \equiv N_{max} \ , \tag{28}$$

Possible cosmological scenario VI

implying that only the fluctuations (domain wall seeds) generated before N_{max} eventually collapse into black holes. The walls whose seeds are produced after N_{max} have their thickness exceeding their Schwarzschild radius.
Let us now estimate maximal mass:

$$m_{max} = 16\pi\Lambda^2 f \left(\frac{e^{2(N_{inf}-N_{cl})}}{2HN_{inf}}\right)^2 \approx 2.3 \cdot 10^{-11} M_{\odot}. \tag{29}$$

Now estimate minimal mass from the condition width of the wall should be less than its Schwarzschild radius:

$$m_{min} > \frac{f}{4G\Lambda^2} \approx 0.5 \cdot 10^{-4} M_{\odot}. \tag{30}$$

In both estimations we have adopted $\Lambda = 10^{-10} GeV$. Since $m_{min} \gg m_{max}$ there is no way to form a black hole in our particular scenario.

Summary

- GB scalar allows to alleviate isocurvature bounds.
- ALP arising during cosmological inflation provides scale-dependent power spectrum.
- Our model provides a possibility to generate fuzzy dark matter particles.
- Black hole production found to be not feasible.

Thank you for your attention!